ACCURACY OF THE LHC OPTICS MEASUREMENT BASED ON AC DIPOLES*

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Abstract

Tight tolerances in the LHC require optics measurement with very good accuracy. Therefore, AC dipoles are employed as the primary devices to measure the LHC optics. The accuracy of the measurement is mainly determined by the length of the coherent signal, signal-to-noise ratio of the measurement, and the data processing to effectively suppress the noise. This paper presents numerical and experimental studies of how these factors affect the accuracy of the LHC optics measurement using the AC dipoles.

INTRODUCTION

AC dipoles are the primary exciter for optics measurements in the LHC [1, 2]. By observing an excited oscillation of a beam bunch with beam position monitors (BPMs), we can make a prompt optics measurement in the ring. The LHC has tight tolerances in various parameters and requires precise measurements and adjustments of them, including linear optics parameters [3]. An excitation produced by the AC dipole does not decohere [4], unlike a kick excitation, and hence is expected to provide measurements with a good accuracy. The accuracy of the optics measurement is determined by signal-to-noise ratio (SNL), length of a data set, and a type of the data processing. This paper studies influences of these three factors on linear optics measurements based on the AC dipole. We compare two algorithms for the data processing, an interpolated FFT refereed to as SUSSIX [5] and the singular value decomposition (SVD) [6, 7].

MODEL

Parameters of interest in this paper are phase advance between two adjacent BPMs, $\Delta\psi$, and β-function at BPMs, β. We use a simple statistical model describing the effect of white noise in the BPM data on measurements of $\Delta\psi$ and β. In the LHC, β is normally determined from the measured $\Delta\psi$ and an optics model [2, 8, 9]. The error in this method can be easily estimated from the error in $\Delta\psi$. Hence, we focus on errors in $\Delta\psi$ and β determined from the amplitude of the excitation. To clarify the effect of noise, we only consider BPMs in the arcs in this paper.

When a bunch is excited with an AC dipole, it's position on n-th turn observed by one BPM is given by

$$x(n) = a\cos(2\pi Q_d n + \psi) + w(n)\,, \tag{1}$$

where n goes up to N, Q_d is the driving tune of the AC dipole, and $w(n)$ describes white noise with a standard deviation (STD) σ_w. SUSSIX and SVD provide the amplitude and phase, a and ψ, from $x(n)$. If the measurement of $x(n)$ is repeated, STD of a and ψ is estimated:

$$\sigma_\psi \simeq \frac{\sigma_a}{a} \simeq \sqrt{\frac{2}{N}}\frac{\sigma_w}{a}\,. \tag{2}$$

We suppose $a_{F(D)}$, $\beta_{F(D)}$, and $\sigma_{\psi,F(D)}$ are the amplitude, β, and STD of the phase measured with BPMs at the locations of the focusing (defocusing) quadrupoles. STDs $\sigma_{\psi,F}$ and $\sigma_{\psi,D}$ satisfy $\sigma_{\psi,D} \simeq r\sigma_{\psi,F}$, where $r = a_F/a_D = (\beta_F/\beta_D)^{1/2} \simeq 2.3$ in the LHC. Given this, STD of $\Delta\psi$, $\sigma_{\Delta\psi}$, is estimated:

$$\sigma_{\Delta\psi} = \sqrt{\sigma^2_{\psi,F} + \sigma^2_{\psi,D}} \simeq \sqrt{\frac{2(1+r^2)}{N}}\frac{\sigma_w}{a_F}\,. \tag{3}$$

For β, we consider the STD normalized with $\beta^{1/2}$:

$$\frac{\sigma_\beta}{\sqrt{\beta}} \simeq \sqrt{\frac{8\beta_F}{N}}\frac{\sigma_w}{a_F} \simeq \sqrt{\frac{8\beta_D}{N}}\frac{\sigma_w}{a_D}\,. \tag{4}$$

Because a is proportional to $\beta^{1/2}$, this parameter is independent of β and we do not need to distinguish the locations of the focusing and defocusing quadrupoles for this parameter.

We treat measurements of $\Delta\psi$ from each BPM pairs and measurements of β from each BPM as repeated measurements of each parameter and consider the distribution of the sample STD[1]. For instance, in a typical optics measurement in the LHC, we repeat a measurement in a given condition for three times. The sample STDs of $\Delta\psi$ and β, $s_{\Delta\psi}$ and s_β, are from three measurements of $\Delta\psi$ and β for each BPM and then we consider the distribution of $s_{\Delta\psi}$ and s_β for all the BPMs in the arcs. The distribution of the sample STDs is described by the χ^2-distribution function:

$$\rho(s) = \frac{2}{2^{(m-1)/2}\Gamma((m-1)/2)\,\bar\sigma}\left(\frac{s}{\bar\sigma}\right)^{m-2} e^{-s^2/(2\bar\sigma^2)}, \tag{5}$$

where s is either $s_{\Delta\psi}$ or $s_\beta/\beta^{1/2}$, m is the number of repeated measurements, $\bar\sigma$ is STD of mean, either $\sigma_{\Delta\psi}/m^{1/2}$ or $\sigma_\beta/(m\beta)^{1/2}$. Please note that STDs $\sigma_{\Delta\psi}$ and $\sigma_\beta/\beta^{1/2}$ are a fit parameter of Eq (5) for a given distribution of the sample STDs. For a measurement in a given condition, $\sigma_{\Delta\psi}$ and $\sigma_\beta/\beta^{1/2}$ can be determined by repeating the measurement and fitting the distribution of the sample STDs to Eq (5).

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[1] We define the sample STD of an observable x for m repeated measurements as $s_x \equiv [\sum_j (x_j - \bar{x})^2/m]^{1/2}$.

SIMULATION

Tracking simulations including the AC dipole are performed for the injection optics of the LHC Beam1 with MADX [10]. Positions of the particle are recorded at BPMs in the arcs and white noise with $\sigma_w = 0.1$ mm, a typical value for the LHC BPM system, is added to the recorded positions. As a measurement data is analyzed [2, 9], the amplitude and phase of the oscillation excited with the AC dipole are extracted from the recorded turn-by-turn position with SUSSIX and SVD and then converted to $\Delta\psi$ and β. Parameters of the AC dipole is set so that the oscillation amplitude at the focusing quadrupoles is 1 mm, which is a typical value for measurements in the LHC, making SNL $\simeq$ 0.1. The amplitude of the AC dipole field is linearly ramped up from zero to the maximum value, kept at the maximum value (plateau), and then linearly ramped down to zero as the real system. During the LHC runs in 2010 and 2011, lengths of the ramp and plateau of the AC dipoles are fixed to 2250 turns (200 ms). Given that the maximum recording length of the LHC BPM system (for the turn-by-turn position measurement) is increased to about 3400 turns in 2011 whereas the plateau length of the AC dipole remains 2250 turns, the effect to include the ramp parts of the AC dipole in the data processing with SUSSIX and SVD is also studied in the simulations.

Figure 1 compares the distributions of the errors (difference between the reconstructed value and the model value) (top) and the sample STDs (bottom) for $\Delta\psi$. Both are calculated from simulated measurements repeated for two, three, and five times. On the top, by definition, STD of the distribution of the errors is the STD of mean and hence is proportional to $m^{-1/2}$: $\bar{\sigma}_{\Delta\psi} = \sigma_{\Delta\psi}/m^{1/2}$. On the bottom, we can see that the distribution of the sample STDs fits well to the χ^2-distribution. As discussed in the previous section, the distribution of the sample STDs provide the STD of mean, $\bar{\sigma}_{\Delta\psi}$, as a fit parameter and hence also

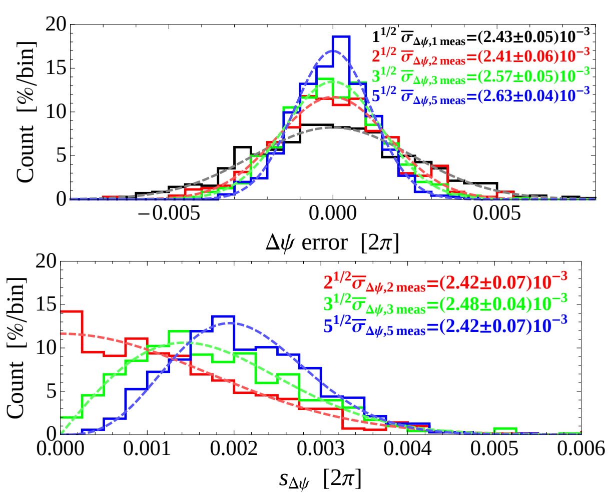

Figure 1: Simulated distributions of the error (top) and sample STD (bottom) of $\Delta\psi$ from SUSSIX.

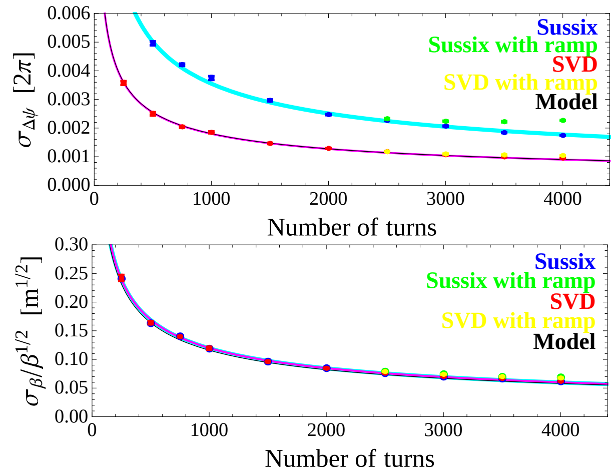

Figure 2: Simulated $\sigma_{\Delta\psi}$ and $\sigma_\beta/\beta^{1/2}$ vs. number of turns, comparing results from SUSSIX and SVD.

provide $\sigma_{\Delta\psi}$. We can see that $\sigma_{\Delta\psi}$ from the distributions of the errors and the sample STDs agree reasonably well.

Figure 2 shows the dependence of $\sigma_{\Delta\psi}$ and $\sigma_\beta/\beta^{1/2}$ on the number of turns. Each data point is from the fit to the distribution of the sample STDs as done in Fig 1. The cyan and magenta curves are the fits proportional to $N^{-1/2}$ to the data points of SUSSIX and SVD. The black curves are the model predictions (Eqs (3) and (4)) for the given SNL $\sigma_w/a_F \simeq 0.1$. Both $\sigma_{\Delta\psi}$ and $\sigma_\beta/\beta^{1/2}$ from both SUSSIX and SVD are almost identical to the model predictions except for $\sigma_{\Delta\psi}$ from SUSSIX. Green and yellow data points are for the cases when the plateau of the AC dipole is 2250 turns but more than 2250 turns are included in the data processing. As we can see, $\sigma_{\Delta\psi}$ from SUSSIX starts to degrade around 3000 turns but others still improve along with the number of turns. However, the improvement is less compared to when the length of the plateau is extended.

MEASUREMENT

To test the model and the result from the simulation, one data set from the injection optics of the LHC Beam1, where a measurement is repeated four times, is analyzed. For these measurements, the SNL at focusing quadrupoles is ∼0.07. Figures 3 and 4 show the distributions of the sample STDs of $\sigma_{\Delta\psi}$ and $\sigma_\beta/\beta^{1/2}$, comparing the results from two, three, and four repeated measurements. The distributions fit reasonably well to the χ^2-distribution as expected. As seen in the simulation, $\sigma_{\Delta\psi}$ from SUSSIX is twice as large as that of SVD whereas $\sigma_\beta/\beta^{1/2}$ from SUSSIX and SVD are nearly identical.

Figure 5 shows dependence of $\sigma_{\Delta\psi}$ and $\sigma_\beta/\beta^{1/2}$ on the number of turns. The same data set as Figs 3 and 4 are used and each data point is calculated as we did in Fig 2. Figure 5 shows the result from three repeated measurements but the result from a different number of repeated measurements remains similar. This is consistent with the re-

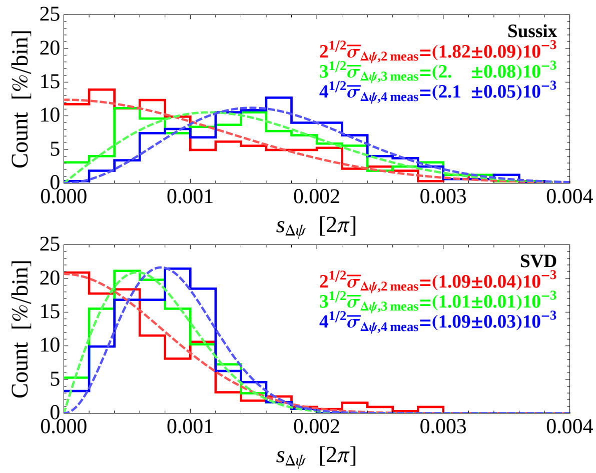

Figure 3: Measured distributions of the sample STDs of $\Delta\psi$, comparing results from SUSSIX and SVD.

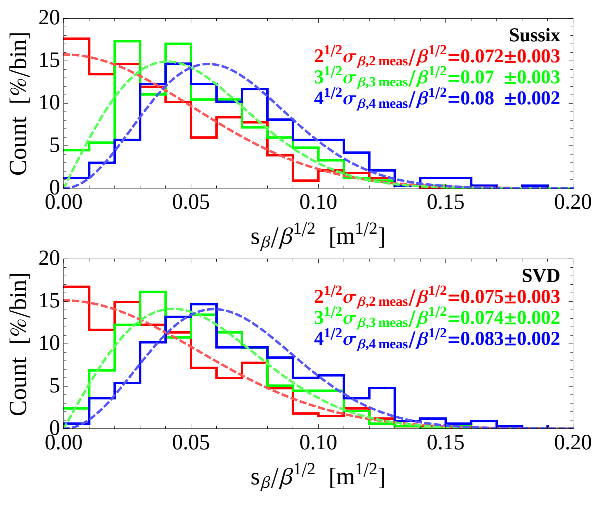

Figure 4: Measured distributions of of the sample STDs of β, comparing results from SUSSIX and SVD.

sult of Figs 3 and 4 where $\sigma_{\Delta\psi}$ and $\sigma_\beta/\beta^{1/2}$ determined from different number of repeated measurements are close to each other. In Fig 5, cyan and magenta curves are the fits proportional to $N^{-1/2}$ to the data points of SUSSIX and SVD. For $\sigma_{\Delta\psi}$, we can see the both results follow well to $N^{-1/2}$. As seen in the simulation, the result from SVD is almost identical to the model based on the given SNL (green) whereas the result from SUSSIX is twice as large compared to that from SVD. For β, we can see deviations from $N^{-1/2}$ dependence for both SUSSIX and SVD. The results from SUSSIX and SVD are still almost identical as seen in the simulation but are slightly worse than the model (green) unlike the simulation. As discussed in the previous sections, the plateau of the AC dipole is 2250 turns so the results of 2500 turns and 3000 turns include data points during the ramp up and ramp down of the AC dipole in the data processing with SUSSIX and SVD. We may see small degradations of $\sigma_{\Delta\psi}$ for the results of 3000 turns for both SUSSIX and SVD but we need data sets with more number of turns to verify this effect.

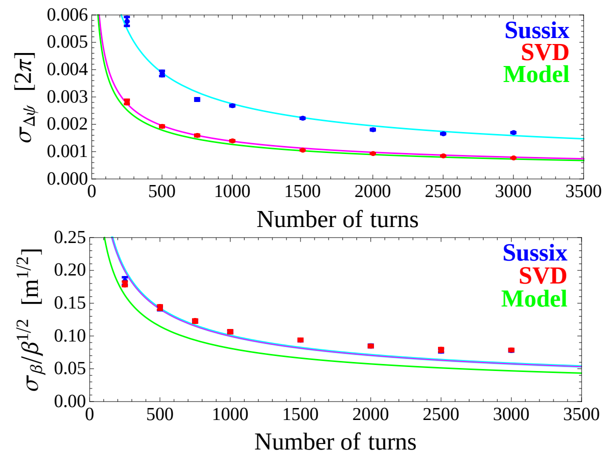

Figure 5: Measured $\sigma_{\Delta\psi}$ and $\sigma_\beta/\beta^{1/2}$ vs. number of turns, comparing results from SUSSIX and SVD.

CONCLUSIONS

Accuracy of the linear optics measurement based on the AC dipole is studied in simulations and experiments and two data processing algorithms, SUSSIX and SVD, are compared. It is demonstrated that the data from the AC dipole processed with SUSSIX or SVD provides the accuracy almost as good expected from a model, except for the phase advance measurement from SUSSIX. In a standard condition of LHC optics measurements, where SNL is ∼0.1 and the date length is 2000 turns, STDs in the phase advance and β-function at focusing quadrupoles are ∼0.001 [2π] and ∼1% for one measurement and the accuracy can be further improve by increasing the excitation amplitude and repeating the measurement. Therefore, the statistical effect is not a problem for linear optics measurements using the AC dipole and the accuracy is limited by the systematic effects, such as BPM gain errors.

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