# METHODS AND TOOLS TO SIMULATE AND ANALYZE NON-LINEAR DYNAMICS IN ELECTRON STORAGE RINGS

Laurent S. Nadolski*
Synchrotron SOLEIL, Gif-sur-Yvette, France

*Abstract*

This paper presents the different approaches and tools that have been recently developed, while trying to understand or predict the non-linear dynamics of electron storage rings. Different algorithms have been lately used at different places to optimize the sextupole tunings, while the refinement of the models of existing machines together with more accurate measurement techniques enable us now to have a much better understanding of the limitations of such facilities especially in the presence of insertion devices.

## INTRODUCTION

Requirements for today and tomorrow's third generation synchrotron light sources (3GLSs) are very challenging in many respects [1]. Optics-wise the route is set for ultra-low emittance rings in order to reach diffracted limited photon beam in both transverse planes. If the lowest horizontal emittance is reached today by Petra-III facility with a value of 1 nm.rad, future 3GLSs will bring the emittance as low as 10 pm.rad (See Tab. 1). Lattices of such facilities are more and more complex based on theoretical minimum emittance (TME), multi bend achromat scheme (MBA) or combination of both of them [2, 3]. Some accelerators make the choice to introduce damping wigglers to reduce furthermore the emittance [4, 5, 6, 7]. So there is a significant shift from simple lattices (see initial ALS [8] lattice with 2 sextupole families) to highly complex multi-parameter lattices. New 3GLSs have typically ten families of quadrupole and sextupole magnets with a trend for individual power supplies. Higher order multipoles start to be introduced from a design stage like in MAX-IV [9].

Table 1: Low Horizontal Emittance for Existing* and Future Synchrotron Light Sources

| Storage Ring | Energy (GeV) | H-emittance |
|---|---|---|
| Petra-III* | 6 | 1 nm.rad |
| PEPX | 4.5 | 0.1 nm.rad |
| NSLS-II | 3 | 0.6 nm.rad |
| MAX-IV | 3 | <0.3, nm.rad |
| SPring8 upgrade | 6 | 10 pm.rad |
| APS upgrade | 7 | 15 pm.rad |

Designing state of the art ultra-low emittance rings needs to fulfill additional requirements to provide high flux and brilliance to the users. Among them, a large horizontal dynamic aperture (typically 15 to 25 mm) is necessary for injection purpose as long as no technical solution is found for on-axis injection. A 2 to 6% transverse momentum aperture needs to be reached to give a large Touschek lifetime, reduce beam losses, activation in the tunnel, and running cost. This latter requirement stands even if 3GLSs run the so-called Top-up injection mode. Mean time between failure of such facilities aims to be 1 week or more. Moreover there is a tendency to provide more and more types of bunch filling patterns, to taylor them to the need of various user communities (flux, brilliance based, time resolved, THz radiation based). Tayloring the bunch filling pattern [10, 11] or the beam properties is a general trend: low momentum compaction factor lattice, combining short and long bunches [12] or even delivering simultaneously several beams [13]. Finally obtaining a high performance lattice is not enough since a light source hosts lots of insertion devices (IDs) with possible impact on the beam performances. These are for instance low gap in-vacuum undulators, Apple-II, -III, Figure-8 undulators, high field wigglers, fast switching electromagnetic undulators. Lattice robustness to ID configurations is an essential feature of 3GLSs giving even more constraints with today low vertical emittance storage ring (SR) and tomorrow's ultra-low horizontal SR.

In this article after a few words about modeling magnets and IDs, we discuss the tools for optimizing the strong focusing lattices of 3GLSs. A special emphasis is given to frequency map analysis, driving term methods, and genetic algorithms. These methods tend to shift our standard ways of tuning non-linear dynamics and have already given promising experimental results.

## MODELING

### *Main Magnets*

Trustable tracking codes are essential in order to have a good modeling of the on- and off-momentum transverse beam dynamics of particle circulating in an accelerator. Non-spurious effect for long term tracking codes is a main feature to predict dynamic aperture (DA), compute resonance strengths or Touschek lifetime. Due to a damping time of a few milliseconds in electron storage rings, tracking over a few thousand of turns is enough providing that stability index such as diffusion index of frequency maps is used. Most of the modern tracking codes, such as MADX/PTC [14], TRACY-III [15], LEGO [16], ELEGANT [17], SAD [18], are based on symplectic integration schemes. Symplectic integrators have been popularized by Ruth and Forest in the 1983–1990 [19, 20]. Then

* nadolski@synchrotron-soleil.fr

Neri introduced the use of Lie Algebra (1988) [21] and Yoshida techniques to build arbitrary high order integrators (1990) [22]. Implicit and explicit integration schemes are possible. More stable and precise scheme were proposed by McLachlan [23] and Laskar [24]. All the classes of integrators have the required properties of long term integration such as energy preservation, bounded errors, phase stability (see Ref. [25] and references therein for more details).

### *Insertion Devices*

Modeling insertion device has slowly shifted from Halbach formalism to more accurate methods due to the complexity of the IDs in term of non-linear fields, short periods, narrow pole widths, aperiodicity, polarization, and lower gaps. Symplectic integration schemes are based either on Taylor expansion of the electromagnetic field or on kick maps [26, 27, 28].

Moreover the assembling of the IDs has significantly changed in order to reduce all non-linear perturbations. Evolutionary algorithms [29] can be used to sort the magnet blocks and minimizing goal functions: first and second order field integrals, normal, skew gradients, phase errors. Special sets of magnets (magic fingers) are usually placed at the ends of the IDs to reduce the impact of higher order field integrals at large transverse amplitudes. Additional active or passive corrections are introduced if required, and can be included in tracking codes.

## TOOLS FOR OPTIMIZATION

### *Turn-by-Turn Data and BPMs*

Thanks to the improvement of BPM electronics, turn-by-turn (TbT) data have become a valuable tool to analyze non-linear beam dynamics. Typically TbT BPM resolution is 10 $\mu m$ for a few milli-ampere stored beam. An additional constraint when carrying out beam based experiments is the beam smearing due to the fact that the kicked beam is a bunch of particles, all with slightly different energies and amplitudes. Each particle encounters different non-linear magnetic fields. As a result the motion is shifted from coherent to incoherent. The direct consequence is that the amplitude of the beam centroid decreases much faster than the damping time, in a few tens or hundreds of turns. Algorithms to precisely determine tune lose their precision [30]. Lines in the frequency domain excited by resonance of order $m$ decohere $m$ times faster than the tune line [31]. Moreover artifacts from the BPM themselves like mixing between turns, cross-talks between RF-channels of the BPM front-end, RF-channel gain or non-linear response due to the asymmetry of the vacuum chamber hosting the BPM make the analysis difficult [32]. All these facts need to be evaluated and corrected carefully before drawing any conclusion from the TbT data analysis of the non-linear motion of the beam.

### *Frequency Map Analysis*

Frequency Map Analysis (FMA) is a refined Fourier technique to build a quasi-periodic decomposition of a signal over time. For accelerators, the first frequencies of the decomposition are the betatron tunes, the others are related to resonance driving terms (RDTs). FMA builds a 2D-mapping between the dynamic aperture and the tune space giving at once a global and detailed view of the transverse beam dynamics. It is characterized by a convergence speed as fast as the inverse of the fourth power of the number of turns used for analysis (when a Hanning windows is introduced as a filter). It enables us to compute the so called diffusion index which can be coded as a color on the DAs and FMAs allowing us to distinguish linear motion from non-linear or chaotic motion, resonances limiting the on- or off-momentum aperture. FMA features are specially adapted to reduce the tracking number of turns in simulation and is suitable for getting useful information even if the experimental data points are limited because of beam smearing [33, 34, 35]. FMA is now widely used from design stage to beam-based measurements [36, 37, 38]. It is worth noting that a very accurate description of the machine model of BESSY-II was obtained and experimentally validated with FMA only after introducing fringe field of dipoles and quadrupoles, systematic errors like octupoles in quadrupoles and decapoles in correctors hosted in sextupoles. This is an example of the very good agreement that can be reached today between model and experiment for on-momentum dynamics [39]. Similarly, agreement for off-momentum dynamics can be as good as a few percents [40, 41] while a factor two was accepted 10 years ago.

## OPTIMIZATION METHODS AND EXPERIMENTAL RESULTS

### *Sextupole Optimization and Resonant Driving Terms*

The standard method for optimizing sextupoles of 3GLSs is based on perturbation approach using resonant driving terms [42, 43]. First and second orders of perturbation theory for sextupole give respectively a total number of 7 and 13 terms to minimize. Usually lattice designers, after choosing a suitable working point, use least-squared like minimization algorithms to optimize the non-linear beam dynamics. Their work is complicated by the complexity of the adjustment between linear and non-linear optics on one hand, and of on- and off-momentum dynamics optimization on the other hand. Even if powerful tools are now available, this work, although proven to be successful, is still not automated, is tedious and requires a lot of manual iterative steps. Results will mainly depend on the weights put on each RDT and the accelerator physicist's experience. In addition, since this optimization is based on perturbation approach, it does not guarantee the best results. Future challenging 3GLSs are required to have extra knobs in order to reduce further RSDs such as non-linear chromaticity or octupolar terms. For instance SLS then MAX-IV have proposed to re-cable unused extra binding of sextupole magnets as sextupoles and skew quadrupoles [44, 45]. A very nice result has been obtained at SLS. The lifetime reduction was recovered for the operation mode, this means an improvement by a factor 2 of the beam lifetime normalized by the stored current.

### *FMA and RDTs*

Thanks to the improvement of the turn-by-turn BPM precision, new methods for minimizing RDTs directly based on TbT data have been proposed and are very promising [46]. For comparing real and model lattices LOCO code [47] has been proven to be very successful for not too large storage rings. It is based on closed orbit data. Similarly, using turn-by-turn data, FMA-like algorithms give amplitude and phase of the spectral lines of the betatron motion that can be used to compare and correct the real accelerator with respect to the model. The knobs are skew quadrupoles, sextupoles, and higher order multipoles. This technique should allow us, in the future, to calibrate the non-linear model and give a full control of the non-linear resonances [30, 31]. A first application at DIAMOND light source has given slight improvement of the lifetime. With better resolution of BPM TbT data, this method will open new doors to characterize and tune the non-linear beam dynamics using the directly beam [44]. BPM resolution is already good enough for tuning betatron coupling using skew quadrupoles and allow low vertical emittance preservation at ESRF [48]. Another application could give a way to online monitor and tune the multipoles of the storage ring [49].

### *Global Analysis*

New methods for optimizing lattices have appeared over the last decades. An example is the GLASS code [50] which stands for Global Linear Analysis of All Stable Solutions. This is adapted for highly periodic lattices when only a few parameters are scanned due to computation time limitation. In the work reported in Ref. [50], a billion lattices were scanned using 3 quadrupole and 2 sextupole families. After one day of computation, one million stable solutions were identified. Then their main properties were computed to build up a large and exhaustive database. Solutions were then sorted by emittance values, tunes, DA sizes, momentum compaction values, and so on. In such a way a global view of all the performances reachable with the ALS lattice was obtained in a very practical manner. Output of this study gave 13 stable areas, among them only 2 were known before, others are promising or even counter-intuitive. For example, it was discovered that ALS could still run while switching off one or two families of quadrupoles. New potential capabilities enhancing the brilliance were suggested with low horizontal beta-function and lower emittance. However this method cannot be directly applied to most of 3GLSs since the parameters space is too large (typically ten families of quadrupoles and sextupoles).

### *Genetic Algorithms*

Genetic algorithms are a very promising solution to the limitation of GLASS. They are based on direct tracking. The focus was first on optimization of the non-linear dynamics. They are attractive because they open new windows of optimization, solutions never thought about before. Moreover beam-based experiments have started to confirm these findings. Multi-Objective Genetics Algorithms (MOGA) applied to accelerator physics are the results of a long and continuous effort, more than ten years, first started at APS and then followed by others: ALS, BNL, ... [51] (see Ref. [52] for a tutorial to MOGA). Thanks to development of clusters and the improvement of computers, access to massive and parallel computation has become affordable.

Definition of suitable objective functions is essential for direct tracking methods. The most popular and successful functions of merit are on- and off-momentum dynamic apertures. Recently including tune-shifts with amplitude has been shown to speed up the algorithm convergence [53]. Addition of other driving terms is not required even if they can help. To reduce the time for computing dynamic aperture different algorithms have been developed. Only a small number of turns is typically used (64 to 128), which has been proven to be enough as a first step to narrow down the number of solutions. As a second step, long term tracking (1 000 to 2000 turns) using FMA is used to select the best and more robust solutions. As reported in Ref. [53], in the case of NSLS-II, starting with population of 6 000, computation of 300 generations takes less than a week on a Sun Grid Engine Cluster made of 96 standard CPUs. Results give large enough dynamic apertures with flat amplitude tune-shift to avoid crossing resonance lines. An interesting results is that small RDTs is a necessary but not a sufficient condition for getting large DAs. Multi-objective optimization is effective and has been shown to be robust to errors. Experimental results at APS give an increase of the beam lifetime by 25 % and 10 % respectively for 24-bunch and hybrid filling patterns used in operation [51]. Amazingly, it is demonstrated that a slight symmetry break of the SR periodicity results in a 25% lifetime improvement without reduction of the injection efficiency (90–100 %) at APS. Application to an intermediate energy 3GLS, namely DIAMOND, has also given an experimental increase of 25% of the lifetime [54]. These results are the proof that genetic algorithms have become mature for optimizing simultaneously on- and off-momentum apertures. A step forward has been reached at ALS with the optimization of both linear and non-linear beam dynamics using MOGA and the diffusion index from FMA as a merit function [55] .

## CONCLUSION

Over the last 20 years tracking codes have become much more trustable. Agreement between real and modeled lattices have shifted from a factor 2 to down a few percents for tune-shifts with amplitude and energy, on- and off-momentum dynamic apertures. This tremendous step forward is mainly due to the significant progress in diagnostics: the turn-by-turn BPM data and the development of fast dedicated kicker magnets for studying non-linear dynamics. Nowadays, accelerator physicists are well equipped for predicting the performance of a lattice and the effect of the introduction of new equipments such as IDs in a storage ring. FMA like techniques give a global view of the non-linear beam dynamics. Promising results have been obtained using RDTs based on turn-by-turn data. This trend will strongly benefit from future higher resolution BPMs. MOGA like algorithms enable us to optimize simultaneously the on- and off-momentum apertures. This is a significant shift for sequential to parallel optimization. These new ways of optimization have opened new horizon windows with sometime unpredictable and amazing results. So the improvement in lattice modeling combined with the latest simulation techniques and progress on accurate beam based measurements using TbT data makes us confident for designing and operating of ever brighter light sources, diffracted limited in both horizontal and vertical planes.

## ACKNOWLEDGEMENT

The author would like to thanks M. Borland, L. Emery, P. Kuske, X. Huang, Y. Cai, J. Safranek, C. Steier, D. Robin, L. Yang, K. Sotome, S. Leemann, A. Streun, R. Bartolini, A. Franchi, R. Tomás, Y. Papaphillipou, A. Kling, and M. Chiu for providing materials for this article and many useful discussions. The author expresses his gratitude to A. Nadji and R. Nagaoka for very stimulating discussions.

## REFERENCES

[1] M. Bei et al., "The Potential of an Ultimate Storage Ring for Future Light Sources", Nucl. Intrum. & Methods A, Vol. 622, Issue 3, pp. 518–535 (2010).

[2] S. C. Leemann and A. Streun, "Perspectives for future light source lattices incorporating yet uncommon magnets", Phys. Rev. ST Accel. Beams, 14:030701, Mar 2011.

[3] Y. Cai. "Single-particle dynamics in electron storage rings with extremely low emittance. Nuclear Instruments and Methods", Nucl. Intrum. & Methods A, Vol. 645(1), pp. 168–174 (2011).

[4] H. Wiedemann, "An Ultra-Low Emittance for PEP using Damping Wigglers", Nucl. Intrum. & Methods A, Vol. 266 (1988).

[5] M. Tischer et al., "Damping Wigglers for the PETRA-III Light Source", PAC'05, Knoxville, pp. 2446–2448, (2005).

[6] S. Ozaki et al., "Philosophy for NSLS-II design with sub-nanometer horizontal emittance", PAC'07, Albuquerque, pp. 77–79 (2007).

[7] S. C. LeemanN et al. "Beam Dynamics and Expected Performance of the Sweden's new Storage-Ring light source: MAX-IV", Phys. Rev. ST Accel. Beams, 12:120701, Dec 2009.

[8] Advanced Light Source 1–2 GeV Synchrotron Radiation Source : Conceptual Design Report, LBNL publication PUB-5172, Berkeley (1986).

[9] MAX-IV Detailed Design Report, available for download at http://www.maxlab.lu.se/maxlab/max4/index.html.

[10] D. S. Robin et al.,"Novel Schemes for Simultaneously Satisfying High Flux and TOF Experiments in a Synchrotron Light Source", Proceedings of EPAC'08, Genoa, pp. 2100–2102 (2008).

[11] G. Portmann et al., "A Potentially New Operational Mode at the ALS: Pseudo Single Bunch", 37th ICFA Advanced Beam Dynamics Workshop on Future Light Sources, Hamburg, Germany, May 2005.

[12] G. Wüstefeld, A. Jankowiak, and M. Ries, "Simultaneous Long and Short Electron Bunches in Bessy II Storage Ring", THPC014, these proceedings.

[13] M. Ries, "Simultaneous Long and Short Electron Bunches in BESSY-II Storage Ring - an Hybrid Mode based on Non-linear Momentum Compaction", TUOAB02, these proceedings.

[14] MAD home page, http://mad.web.cern.ch/mad.

[15] J. Bengtsson, Tracy-2 User's Manual (unpublished).

[16] Y. Cai et al., "LEGO: A Modular Accelerator Design Code", SLAC-PUB-7642, August 1997.

[17] M. Borland, "Users Manual for elegant", APS LS-231, May 6, 1993.

[18] SAD home page, http://acc-physics.kek.jp/SAD/

[19] R. D. Ruth, "A canonical Integration Technique", IEEE Trans. Nucl. Sci., Ns-30, pp. 2669–2671 (1983).

[20] E. Forest and R. D. Ruth, "Fourth-order symplectic integration", Physica D, Nol. 43(1), pp. 105–117 (1990).

[21] F. Neri, "Lie algebras and canonical integration", Dept. of Phys., Maryland University (1987).

[22] H. Yoshida, "Construction of high order symplectic integrators", Phys. Let. A, vol. 150, No 5–7, pp. 262–268 (1990).

[23] R. I. McLachlan, "Composition methods in the presence of small parameters", BIT Vol. 35, No. 2, pp. 258–268 (1995).

[24] J. Laskar and P. Robutel, "High order symplectic integrators for perturbed Hamiltonian systems", Celestial Mechanics, Vol. 80, pp. 39–62 (2001).

[25] E. Forest, "Geometric integration for particle accelerators" , J. Phys. A: Math. Gen. 39 5321(2006).

[26] P. Elleaume et al., "A new approach to the electron beam dynamics in undulators and wigglers", EPAC'92, pp. 661–663 (1992)

[27] Y. K. Wu, E. Forest, and D. S. Robin, "Explicit symplectic integrator for s-dependent static magnetic field" Phys. Rev. E, 68:046502, Oct 2003.

[28] J. Bahrdt and G. Wstefeld, "Symplectic tracking and compensation of dynamic field integrals in complex undulator structures", 10.1103/PhysRevSTAB.14.040703 (2011).

[29] O. Rudenko and O. Chubar, "An Evolutionary Approach to Shimming Undulator Magnets for Synchrotron Radiation Sources", Proceedings of 9th Int. Conf. on Parallel Problem Solving from Nature PPSN IX, p.362 (2006).

[30] R. Bartolini et al., "Precise Measurement of The Betatron Tune", Part. Acc. Vol. 55, pp. 247–256 (1995).

[31] R. Tomás, "Direct Measurement of Resonance Driving Terms in the Super Proton Synchrotron of CERN using Beam Position Monitors', 'Ph.D. Thesis Dissertation (2002).

[32] R. W. Helms and G. H. Hoffstaetter, "Orbit and optics improvement by evaluating the non-linear beam position monitor response in the cornell electron storage ring", Phys. Rev. ST Accel. Beams, 8:062802, Jun 2005.

[33] J. Laskar, "Secular evolution of the solar system over 10 million years", Astronomy & Astrophysics, Vol.198, pp. 341–362 (1988).

[34] J. Laskar, "The chaotic motion of the solar system. A numerical estimate of the size of the chaotic zones", Icarus, Vol. 88, pp. 266–291 (1990).

[35] J. Laskar, "Introduction to Frequency Map Analysis", Proceedings of 3DHAM95 NATO Advanced Institute, S'Agaro, June 1995, C. Simó Ed., pp. 134–150.

[36] D. S. Robin et al., "Global Dynamics of the Advanced Light Source Revealed through Experimental Frequency Map Analysis", Phys. Rev. Lett. 85, 558–560 (2000) .

[37] C. Steier et al., "Measuring and optimizing the momentum aperture in a particle accelerator", Phys. Rev. E, 65, 056506 (2002).

[38] L. Nadolski and J. Laskar, "Review of single particle dynamics for third generation light sources through frequency map analysis", Phys. Rev. ST Accel. Beams 6, 114801 (2003).

[39] P. Kuske, "Effects of Fringe Fields and Insertion Devices revealed through experimental Frequency Map Analysis", Proceedings of PAC'05, Knoxville, pp. 266-268 (2005).

[40] P. Brunelle et al., "Non linear beam dynamics studies at SOLEIL using experimental frequency map analysis", Proceedings of IPAC'10, Japan, pp. 4653–4655 (2010).

[41] R. Bartolini et al., "Calibration of the non-linear ring model at the diamond light source", Phys. Rev. ST Accel. Beams, 14:054003, May 2011.

[42] J. Bengtsson, "Non-linear transverse dynamics for storage rings with applications to the Low-Energy Antiprotron Ring (LEAR) at CERN", CERN 88-05, August 1988.

[43] J. Bengtsson, "The sextupole scheme for the Swiss Light Source (SLS): An Analytic approach", SLS-Note 9/97, March 1997.

[44] A. Streun, "Nonlinear Beam Dynamics Studies at the SLS", Proceedings of Non-linear Beam Dynamics Workshop, Diamond, 2009, http://www.diamond.ac.uk/Home/Events/Past_events/NBD_workshop.html.

[45] A. Streun, "Non-linear dynamics at the SLS Storage Ring", Proceedings of the Low Emittance Rings Workshop 2010, http://ler2010.web.cern.ch.

[46] R. Bartolini and F. Schmidt. "Non-linear ring model calibration with frequency analysis of betatron oscillations". Proceedings of PAC'05, Knoxville, pp. 1452–1454 (2005).

[47] J. Safranek, "Experimental Determination of Storage Ring Optics using Orbit Response Measurements", Nucl. Inst. And Meth. A388, 27 (1997).

[48] A. Franchi et al, "Vertical emittance reduction and preservation in electron storage rings via resonance driving terms correction", Phys. Rev. ST Accel. Beams, 14:034002 (2011).

[49] A. Franchi, R. Tomás, and F. Schmidt, "Magnet strength measurement in circular accelerators from beam position monitor data", Phys. Rev. ST Accel. Beams, 10:074001 (2007).

[50] D. S. Robin et al., "Global analysis of all linear stable settings of a storage ring lattice", Phys. Rev. ST Accel. Beams, 024002 (2008).

[51] M. Borland et al, "Direct Method of Optimization of Storage Ring Dynamic and momentum Aperture", Proceedings of PAC'09, Vancouver, pp. 3850–3852 (2009).

[52] A. Konak et al, "Multi-objective optimization using genetic algorithms: A tutorial", Reliability Engineering and system Safety, Volume 91, Issue 9, pp. 992–1007 (2006), ISSN 0951-8320, 10.1016/j.ress.2005.11.018.

[53] L. Yang, Y. Li, W. Guo, and S. Krinsky. "Multiobjective optimization of dynamic aperture", Phys. Rev. ST Accel. Beams, 14:054001 (May 2011).

[54] Courtesy of Ricardo Bartolini.

[55] C. Sun, "Dynamics Aperture Optimization using Genetics Algorithms", Proceeding of PAC'11, New York (2011).