OPTICS CONSIDERATIONS FOR THE DELAY LOOP IN THE CLIC DAMPING RINGS COMPLEX

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Abstract

For the recombination of the two trains coming from the CLIC damping rings, a delay loop will be used in order to obtain the nominal 0.5 ns bunch spacing. The optics design of the loop is based upon an isochronous ring, in order to preserve the longitudinal beam distribution. Analytical expressions for achieving isochronous conditions in high order for Theoretical Minimum Emittance cells are obtained. A parametrisation of the quadrupole settings for achieving these conditions is presented, along with general considerations regarding the choice of bending magnet characteristics.

INTRODUCTION

A unique loop, downstream of the CLIC damping rings, is used to recombine consecutively both particle species' trains, in order to form the bunch pattern with 0.5 ns time structure required by the collider. This delay loop (DL) should be designed in such a way so as to preserve the beam characteristics achieved in the damping rings making train recombination as transparent as possible for the collider performance. One important condition for keeping the longitudinal beam distribution unperturbed is that the ring is isochronous, i.e. its the path length is independent of the energy. Isochronicity conditions require that the optics is tuned so as to eliminate the momentum compaction factor. Although the leading order contribution to the momentum compaction factor stems from the average dispersion function in the dipoles, the remaining higher order terms depend on the dispersion evolution at all orders along the whole cell. The aim of this paper is to analyse, understand and consequently optimise the cell parameters that lead to high-order isochronicity conditions, considering only linear transverse magnetic fields. The high-order dispersion propagation, necessary for the calculation of the momentum compaction factor relations, is obtained analytically for a Theoretical Minimum Emittance (TME) cell, for which a detailed optics analysis has been already worked out, in thin element approximation [1]. Higher order momentum compaction factors are calculated and their dependence from the quadrupole strengths is illustrated. Finally, the impact of this optics to the horizontal emittance and energy loss due to synchrotron radiation is discussed.

HIGH ORDER DISPERSION FUNCTIONS

In order to analytically calculate the momentum compaction factor of the entire TME cell, it is essential to estimate the dispersion function up to high order, e.g. two orders higher than the leading one. This is done by propagating and solving the equations of the dispersion for every element of the cell, using the thick lens expressions. As a first step, only transverse fields for dipoles and quadrupoles are considered, without any linear or non-linear errors. In order to analytically find the solutions of the element strengths as a function of the high order momentum compaction factor, thin lens approximations can be applied, leading to polynomial expressions.

In Fig. 1, the layout of the cell is shown, with the dipole D of length l_{dip}, drift spaces s_1, s_2, s_3 and two quadrupoles Q1 and Q2, of focal length f_1 and f_2, respectively.

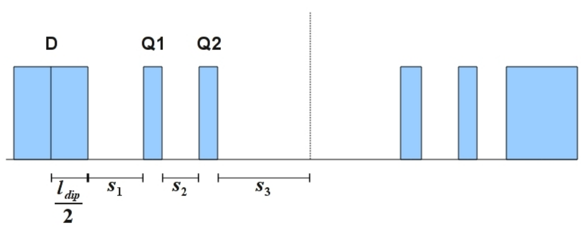

Figure 1: Layout of a TME cell.

The propagation of the dispersion function starts from the centre of the dipole, where the derivative of the dispersion is zero [1] until the point of symmetry at the end of the drift space s_3, where also the derivative is zero. Under these considerations, a set of solutions for the dispersion equations up to second order for every element of the TME cell can be obtained.

The analytic form of the differential equations of the dispersion can be calculated [2] from the Hamiltonian of the off-momentum particle with a momentum deviation $\delta = \frac{p-p_0}{p_0}$, where p_0 is the nominal momentum.

Assuming that the delay loop has no vertical bending,, i.e. $\frac{1}{\rho_y} = 0$, and considering motion along the horizontal x axis only, the canonical Hamilton equations in that plane can be used to derive the equations of motion for off-momentum particles. Considering only transverse magnetic fields for separated function magnets, the longitudinal vector potential component for a bending and quadrupole magnet are:

$$\frac{e}{p_0}A_s^{bend} = -\frac{1}{2}\left(1+\frac{x}{\rho_x}\right) \quad , \tag{1}$$

$$\frac{e}{p_0}A_s^{quad} = -\frac{1}{2}g_0 x^2 \quad , \tag{2}$$

where $g_0 = \frac{e}{p_0}(\frac{\partial B_y}{\partial x})\Big|_{x=y=0}$ is the normalised quadrupole gradient. Using the expansions up to third order of the particle's horizontal position with respect to δ and its derivative with s:

$$x = \eta_0\delta + \eta_1\delta^2 + \eta_2\delta^3 \ , \qquad (3)$$
$$x' = \eta_0'\delta + \eta_1'\delta^2 + \eta_2'\delta^3 \ , \qquad (4)$$

with η_0, η_1, η_2 the zero, first and second order dispersion functions, respectively, and x', η_0', η_1', η_1' the derivatives of x, η_0, η_1, η_2 with respect to s, the following differential equations up to second order, for the dispersion, can be derived [2]:

$$\begin{aligned}
&\eta_0'' + (\tfrac{1}{\rho_x^2} + g_0)\eta_0 = \tfrac{1}{\rho_x} \\
&\eta_1'' + (\tfrac{1}{\rho_x^2} + g_0)\eta_1 = g_0\eta_0 - \tfrac{1}{\rho_x}(1 - \tfrac{1}{2}\eta_0'^2) \\
&+\tfrac{2}{\rho_x^2}\eta_0 - \tfrac{1}{\rho_x^3}\eta_0^2 \\
&\eta_2'' + (\tfrac{1}{\rho_x^2} + g_0)\eta_2 = g_0(\eta_1 - \eta_0 - \tfrac{3}{2}\eta_0\eta_0') \\
&+\tfrac{1}{\rho_x}(1 + \eta_1'\eta_0' + \tfrac{3}{2}\eta_0'^2) + \tfrac{2}{\rho_x^2}(\eta_1 - \eta_0 - \eta_0\eta_0'^2) \\
&-\tfrac{1}{\rho_x^3}(2\eta_1 - \eta_0)\eta_0
\end{aligned} \qquad (5)$$

MOMENTUM COMPACTION FACTOR CALCULATIONS AND ANALYSIS

Zero Order Momentum Compaction Factor

The zero order momentum compaction factor can be calculated using the solution of the dispersion inside the dipole:

$$\eta_0 = \rho_x + (\eta_c - \rho_x)\cos(\frac{s}{\rho_x}) \qquad (6)$$

Using the relation for the zero order momentum compaction factor

$$\alpha_0 = \frac{1}{L}\int_0^L \frac{\eta_0}{\rho_x} ds$$

where L is the length of the dipole, Eq.(6) can be used to calculate α_0. This result can be simplified if thin lens approximation is applied, resulting to:

$$\alpha_0 = \frac{24\eta_c - (\eta_c - \rho_x)\theta^2}{12\rho_x} \qquad (7)$$

Solving Eq.(7) with respect to η_c, the dispersion at the centre of the dipole which eliminates the zero order momentum compaction factor is indeed negative:

$$\eta_c = -\frac{7\theta^2\rho}{24} \qquad (8)$$

The behaviour of the zero order momentum compaction factor in the TME cell is shown in Fig. 2 with respect to the bending radius ρ_x and the bending angle θ using a colour scale. The energy is $E = 2.86$ GeV, and for a maximum bending field choice of $B = 0.5$ T, the minimum bending radius becomes $\rho_{xmin} \approx 19m$. The plots correspond to $\rho_x \geqslant \rho_{xmin}$. From Eq.(7), and setting $\eta_c \rightarrow 0$, α_0 is independent of ρ_x and quadratic with θ. This is shown in the left plot of Fig. 2, where a very small η_c value is chosen. When η_c gets large negative values, the trend is inverted and at least for small bending angles, α_0 scales as $1/\rho_x$, as shown in the right plot of Fig. 2.

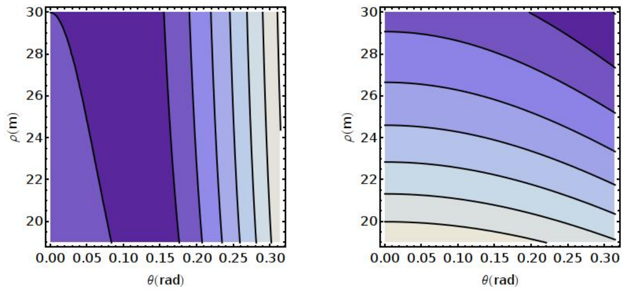

Figure 2: α_0 with respect to bending radius ρ_x and bending angle θ, for $\eta_c = -0.015m$ (left) and $\eta_c = -0.8m$ (right). Darker areas represent smaller absolute values of α_0.

High Order Momentum Compaction Factor

Expanding the momentum compaction factor up to second order with respect to δ, yields:

$$\alpha_c = \alpha_0 + \alpha_1\delta + \alpha_2\delta^2 \qquad (9)$$

The general equation of the momentum compaction factor is:

$$\alpha_c\delta = \frac{1}{C_0}\oint_C (dl - dl_0) \qquad (10)$$

where C_0 the nominal closed orbit of the synchronous particle, C is the closed orbit of the off-momentum particle, dl the infinitesimal path length of an off-momentum particle which is given by:

$$dl = \sqrt{(1 + \frac{x}{\rho_x})^2 + (x')^2 ds} \qquad (11)$$

and dl_0 the nominal one, which can be calculated by setting $x = 0$ in Eq.(11). Using the expansions (4), $dl - dl_0$ can be computed. Inserting this in Eq.(10) and equating with Eq.(9), the analytic formulas for high orders terms can be derived:

$$\alpha_1 = \frac{1}{L}\int_0^L \left(\frac{\eta_0^2}{2\rho_x^2} + \frac{\eta_1}{\rho_x} + \frac{1}{2}\eta_0'^2\right) \mathrm{d}s \qquad (12)$$

$$\alpha_2 = \frac{1}{L}\int_0^L \left(\frac{\eta_0\,\eta_1}{\rho_x^2} + \frac{\eta_2}{\rho_x} + \eta_0'\,\eta_1'\right) \mathrm{d}s \qquad (13)$$

where α_1 and α_2 are the first and second order momentum compaction factor respectively and L is the length of the element. Inserting the dispersion solutions for the TME cell in the above equations α_1, α_2 and α_c can be calculated from (9).

Fig. 3, shows α_1 and α_2 with respect to f_1, f_2. The same colour code is used as in the previous plots. The white area in the middle of the plots corresponds to quadrupole strengths that do not provide any solutions. One should exclude also areas with both positive or negative quad strengths, as for these values there is no optics stability in both planes. Larger absolute values of f_1, f_2, i.e. weaker focusing is needed for obtaining lower values for α_1 and α_2.

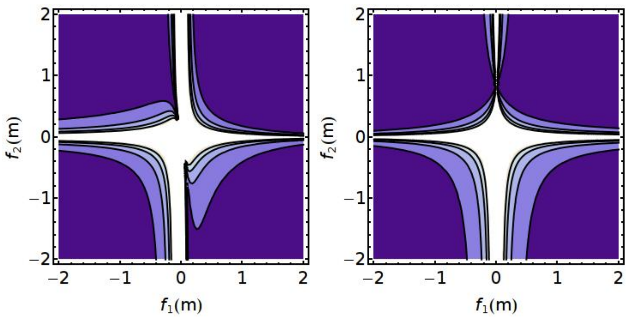

Figure 3: $\alpha_1\delta$ (left) and $\alpha_2\delta^2$ (right) with respect to f_1,f_2. Darker areas represent smaller absolute values of α_0. White areas represent the set of values for f_1 and f_2 that $\alpha_1\delta$ and $\alpha_2\delta^2$ can get no values.

From Eq. (9), the momentum compaction factor of the entire cell, a_c, can be calculated. On the left of Fig. 4 α_c of the TME cell with respect to f_1 and f_2 is shown. A combination of large negative f_2 and a relatively low positive f_1, could lead to small values of α_c.

The plot on the right shows values for positive f_1 and negative f_2 for which α_c is zero.

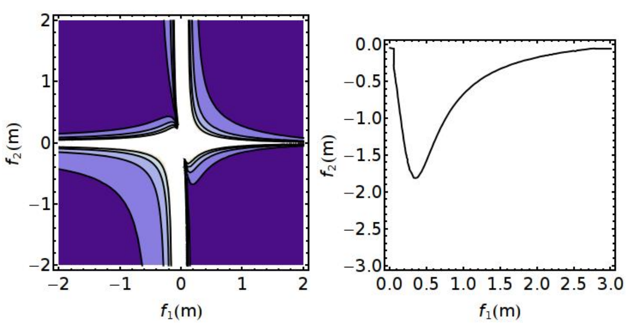

Figure 4: α_c of the TME cell with respect to f_1,f_2 (left) and the curve for sets of values of f_1, f_2 that are solutions for α_c =0 (right).

ENERGY LOSS PER TURN AND EMITTANCE SPREAD

Due to synchrotron radiation, the particles inside the bends lose energy, inversely proportional to the bending radius. On the left plot of Fig. 5, the energy loss δE dependence on ρ is shown, indicating that a larger bending radius is necessary (weaker bending field) for minimizing the energy loss, which should be recovered by an RF cavity with a weak gradient.

In addition, in the delay loop, the perturbation of the horizontal emittance should be minimum. The TME cell, has a specific expression for the horizontal emittance [1]:

$$\epsilon_x = \frac{C_q\gamma_r^2}{J_x\rho_x}\left[\frac{1}{\beta_c}\left(\eta_c^2 - \frac{\theta^2\eta_c\rho_x}{12} + \frac{\theta^4\rho_x^2}{320}\right) + \frac{\theta^2\beta_c}{12}\right]$$

where $C_q = 3.84 \cdot 10^{-13}m$, γ_r is the relativistic gamma factor, J_x the damping partition number, ρ_x the bending radius, β_c and η_c the beta and dispersion functions at the dipole's centre and θ the bending angle of the dipole. Considering constant bending radius, and by differentiating ϵ_x with respect to θ yields:

$$\frac{\delta\epsilon_x}{\epsilon_x} = \left(\frac{4\theta^2(40\beta_c^2 + \rho_x(-40\eta_c + 3\theta^2\rho_x))}{960\eta_c^2 + 80\beta_c^2\theta^2 - 80\eta_c\theta^2\rho_x + 3\theta^4\rho_x^2}\right)\frac{\delta\theta}{\theta}$$

which is the relative deviation of the horizontal emittance with respect to the bending angle relative deviation. The right plot of Fig. 5 shows the behaviour of $\frac{\delta\epsilon_x}{\epsilon_x}$ with respect to θ, for a choice of $\frac{\delta\theta}{\theta} = 10^{-4}$. It is already straightforward from the above relationship, that small values of θ, that is a bigger number of short dipoles with weak field are beneficial, for minimizing both energy loss per turn and the horizontal emittance distortion.

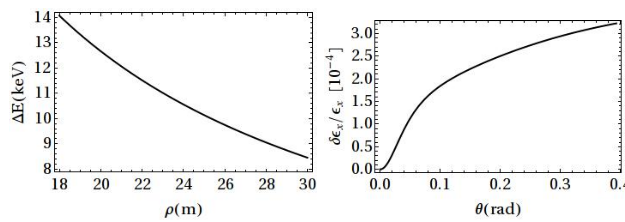

Figure 5: ΔE with respect to ρ (left) and $\frac{\delta\epsilon_x}{\epsilon_x}$ with respect to θ (right).

CONCLUSIONS

The TME cell parameters can be chosen so that the momentum compaction factor is minimised or eliminated and at the same time to be in accordance with a minimal energy loss and horizontal emittance conservation. In particular, it is shown that for low quadrupole strength, the high order momentum compaction factors can be minimised. However, full isochronicity conditions, i.e. minimisation of the leading order momentum compaction factor requires a small negative dispersion at the centre of the dipole, i.e. strong quadrupole strengths. In this respect, the optimal quadrupole values are found for minimising the momentum compaction factor at all orders. In addition, a relatively low magnetic field (high bending), minimises the energy loss combined with a choice of small θ (large number of short dipoles), which would lead to a decrease of $\frac{\delta\epsilon_x}{\epsilon_x}$. We should point out that no non-linear field components, including magnet fringe fields were considered. This would be the next step of this study. Even at this stage, though, this analysis gives us confidence that there is a large parameter space for achieving the optics design for isochronous delay loop, with minimal perturbation of the beam distribution

REFERENCES

[1] F. Antoniou,Y. Papaphilippou, Analytical considerations for linear and non-linear optimization of TME cells. Application to the CLIC pre-damping rings, preprint.

[2] H. Tanaka, et al., A perturbative formulation of non-linear dispersion for particle motion in storage rings, NIMA, 431:3, p.396-408