SELF-CONSISTENT TIME-DEPENDENT QUASI-3D MODEL OF MULTIPACTOR IN DIELECTRIC-LOADED ACCELERATING STRUCTURES*

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Abstract

In this paper the authors present a new quasi-three-dimensional model of multipactor in dielectric-loaded accelerator structures. The model takes into account the axial motion of the particles that was ignored in previous two-dimensional studies. It also includes a self-consistent calculation of multipactor-induced RF power loss.

INTRODUCTION

Multipactor (MP) is known as the rapid growth of the number of secondary electrons emitted from a solid surface in vacuum in the presence of the RF field. The secondary electrons appear due to impacts of energetic primary electrons accelerated by that field. MP occurs in various RF and microwave systems and usually degrades their performance. In particular, it generates RF noise, reduces RF power flow, changes device impedance, stimulates RF breakdown, etc. Therefore, theoretical and experimental studies of MP are of great interest to researchers working in related areas of physics and engineering. In this paper we study the MP in dielectric-loaded accelerator (DLA) structures. The starting point for this work was theoretical and experimental studies of such structures jointly done by Argonne National Laboratory (ANL) and Naval Research Laboratory (NRL) [1-3]. In the theoretical model developed during those studies the space-charge field created by secondary electrons was introduced as an external parameter. However, such approach could result in inaccurate calculations of the MP saturation levels. To address that problem we developed a non-stationary two-dimensional model of MP in which the space charge effects were taken into account self-consistently [4]. We compared results predicted by that model with the experimental data obtained during extensive studies of DLA structures done by ANL, NRL, SLAC National Accelerator Laboratory and Euclid TechLabs, LLC [5]. Good agreement between the two was demonstrated for tubes of large diameter with alumina and quartz liners. However, for tubes with quartz liners of smaller diameter a discrepancy between theory and experiment was observed [6]. There could be several reasons for such disagreement. First, we were assuming that MP was driven mainly by the radial component of the RF electric field. Correspondingly, in our 2D model we were neglecting the axial motion of electrons that in some cases could be making a substantial contribution to the total energy of particles. Secondly, it was assumed that the space charge distribution of secondary electrons was azimuthally uniform which was allowing one using a simple model to calculate the space charge fields. We performed a simple 3D analysis [7-8] to check whether those assumptions were valid. It demonstrated that particles can have a substantial axial velocity component that contributes significantly to their total kinetic energy. It also showed that in some cases the space charge distribution can be non-uniform, however we observed such behaviour only at the initial stage of MP development. Therefore, we decided to leave the space charge field calculation scheme practically the same as in our 2D studies with only slight modifications. Below we present our new 3D model of MP in which the axial motion of the particles is taken into account. We also supplemented it with the self-consistent calculations of the MP-induced attenuation of the RF wave.

MODEL DESCRIPTION

Fig. 1 schematically shows a cross-section of a DLA structure. In this figure, 1, 2 and 3 indicate the vacuum region, dielectric liner and metal wall, respectively, *a* and *b* are the inner and outer radii of the liner.

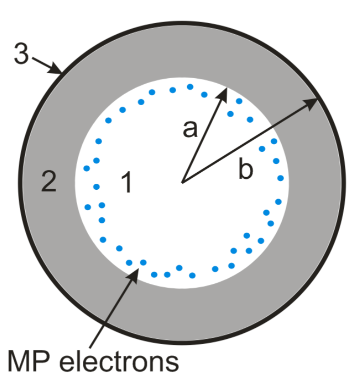

Figure 1: Cross-section of a DLA structure. 1- vacuum region, 2 – dielectric liner, 3 – metal wall.

We are considering non-relativistic particle motion in arbitrary RF and space-charge fields in the vacuum region. The set of corresponding equations of motion in normalized variables in cylindrical coordinates can be written as

$$\frac{d\tilde{v}_r}{d\tilde{t}} = \frac{\tilde{v}_\phi^2}{\tilde{r}} - \left(\tilde{E}_{RF,r} + \tilde{E}_{SC,r} + \tilde{v}_\phi \tilde{H}_{RF,z} - \tilde{v}_z \tilde{H}_{RF,\phi}\right) \quad (1)$$

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$$\frac{d\tilde{v}_\phi}{d\tilde{t}} = -\frac{\tilde{v}_r\tilde{v}_\phi}{\tilde{r}} - \left(\tilde{E}_{RF,\phi} + \tilde{E}_{SC,\phi} + \tilde{v}_z\tilde{H}_{RF,r} - \tilde{v}_r\tilde{H}_{RF,z}\right) \quad (2)$$

$$\frac{d\tilde{v}_z}{d\tilde{t}} = -\left(\tilde{E}_{RF,z} + \tilde{E}_{SC,z} + \tilde{v}_r\tilde{H}_{RF,\phi} - \tilde{v}_\phi\tilde{H}_{RF,r}\right) \quad (3)$$

$$\frac{d\tilde{r}}{dt} = \tilde{v}_r \quad (4)$$

$$\frac{d\tilde{\phi}}{d\tilde{t}} = \frac{\tilde{v}_\phi}{\tilde{r}} \quad (5)$$

$$\frac{d\tilde{z}}{d\tilde{t}} = \tilde{v}_z \quad (6)$$

Here $\tilde{r} = r/a, \tilde{\phi} = \phi, \tilde{z} = z/a$ are the normalized radial, angular and axial particle coordinates, respectively, $\tilde{v}_r = v_r / a\omega$, $\tilde{v}_\phi = v_\phi / a\omega$ and $v_z = v_z / a\omega$ are the corresponding normalized particle velocity components, where $\omega = 2\pi f$ is the wave frequency. Also, in the equations above, $\tilde{E}_{RF,r}$, $\tilde{E}_{RF,\phi}$, and $\tilde{E}_{RF,z}$ are normalized electric components and $\tilde{H}_{RF,r}$, $\tilde{H}_{RF,\phi}$ and $\tilde{H}_{RF,z}$ are the normalized magnetic components of the RF field. The three normalized components of the space charge electric field are denoted as $\tilde{E}_{SC,r}$, $\tilde{E}_{SC,\phi}$ and $\tilde{E}_{SC,z}$. Both RF and space charge electric field components are normalized as follows: $\tilde{E}_{RF,SC} = eE_{RF,SC} / a\omega^2 m$, where e and m are the electron charge and mass, respectively. The RF magnetic field components are normalized as $\tilde{H}_{RF} = e\mu_0 H_{RF} / \omega m$, where μ_0 is the magnetic permeability of free space. We consider the motion of electrons in the RF field of TM_{01} mode only. Its normalized components for the forward wave can be given as:

$$\tilde{E}_{RF,r} = -\alpha \frac{\rho_z}{\rho_1} I_1(\rho_1\tilde{r})\sin(\tilde{t} - \rho_z\tilde{z} + \varphi) \quad (7)$$

$$\tilde{E}_{RF,z} = \alpha I_0(\rho_1\tilde{r})\cos(\tilde{t} - \rho_z\tilde{z} + \varphi) \quad (8)$$

$$\tilde{H}_{RF,\phi} = -\alpha \frac{\tilde{\omega}^2}{\rho_1} I_1(\rho_1\tilde{r})\sin(\tilde{t} - \rho_z\tilde{z} + \varphi) \quad (9)$$

Here I_0 and I_1 are the modified Bessel functions of the first kind of order 0 and 1, respectively, $\alpha = eE_{RF,0} / a\omega^2 m$ is the normalized RF amplitude and $\tilde{t} = \omega t$ is the normalized time. Also, in these expressions, $\rho_1 = |k_{\perp 1}| a$ and $\rho_z = k_z a$ are, respectively, the normalized transverse and axial wavenumbers in the vacuum region and $\tilde{\omega} = \omega a / c$ is the normalized frequency, where c is the speed of light. The normalized wavenumbers can be found from the dispersion equation for TM_{01} mode [4]:

$$\rho_1 \frac{I_0(\rho_1)}{I_1(\rho_1)} = \frac{\rho_2 r'}{\varepsilon_d} \frac{J_0(\rho_2 r')Y_0(\rho_2) - J_0(\rho_2)Y_0(\rho_2 r')}{J_1(\rho_2 r')Y_0(\rho_2) - J_0(\rho_2)Y_1(\rho_2 r')} \quad (10)$$

In this equation, J_0 and J_1 are Bessel functions of the first kind of order 0 and 1, respectively, Y_0 and Y_1 are Bessel functions of the second kind of order 0 and 1, $r' = a/b$ is the ratio of inner and outer radii of the liner and $\rho_2 = k_{\perp 2} b$ is the normalized transverse wavenumber in the dielectric liner. The transverse and axial wavenumbers are related as

$$\rho_1 = \left(\rho_z^2 - \tilde{\omega}^2\right)^{\frac{1}{2}}, \quad \rho_2 = \frac{1}{r'}\left(\tilde{\omega}^2\varepsilon_d - \rho_z^2\right)^{\frac{1}{2}} \quad (11)$$

where ε_d is the relative permittivity of the dielectric material.

As mentioned in the Introduction, at this point of our analysis we are using a simple cylindrical model for space charge field calculation which assumes azimuthal symmetry. It is similar to the space charge model described in Ref. 4.

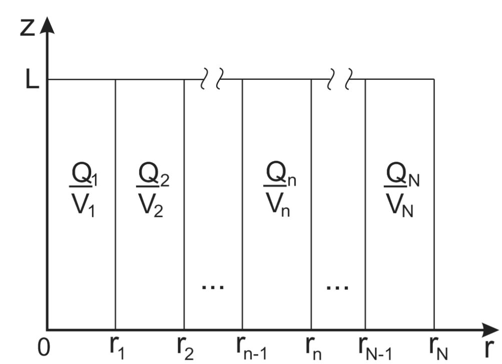

Figure 2: Cylindrical model for the space charge field calculation. N is the number of segments, r_n is the outer radius of the n^{th} layer, where $r_N = a$.

We divide the vacuum region into N segments along radial direction (see Fig. 2) and assume that the volume space charge density Q_n/V_n is constant in each segment. Then, by using Gauss' law, one may easily find the space charge electric field in the n^{th} segment:

$$E_{SC,r} = \frac{Q_n r}{2V_n\varepsilon_0}, \quad n = 1 \quad (12)$$

$$E_{SC,r} = \frac{1}{2\varepsilon_0}\left(\frac{1}{r}\sum_{m=2}^{n} r_{m-1}^2\left(\frac{Q_{m-1}}{V_{m-1}} - \frac{Q_m}{V_m}\right) + \frac{rQ_n}{V_n}\right), n \geq 2. \quad (13)$$

In these expressions, r_n is the outer radius of the n-th layer, Q_n is the total charge contained in this layer, V_n is its volume. One may calculate Q_n by using the following expression:

$$Q_n = q_0 \sum_{k=1}^{K} w_k \,, \tag{14}$$

where K is the total number of particles in the layer, w_k is the particle weight and q_0 is the initial (seed) charge assigned to particles when they appear on the surface of the dielectric. The space charge field component given by expressions 12 and13 should use the same normalization as introduced after expression 6.

We assume that secondary particles have random initial energies E_0 and emission angles θ_e obeying the following distribution functions:

$$f(E_0) = \frac{E_0}{E_{0m}^2} \exp\left(-\frac{E_0}{E_{0m}}\right) \tag{15}$$

$$g(\theta_e) = \frac{1}{2}\sin(\theta_e), \quad 0 < \theta_e < \pi \,, \tag{16}$$

where E_{0m} is the energy corresponding to the maximum of the distribution function $f(E_0)$. We assume that distribution functions for the emission angles are the same in ϕ-r and r-z planes. Upon leaving the surface, the particles continue their motion in the vacuum region until they impact the dielectric wall. Then, based on their impact energy and impact angle, the secondary emission yield is calculated. In our studies we use the well-known Vaughan empirical model for such calculations [9].

To calculate the wave attenuation induced by MP, we divide the vacuum region into M segments in the axial direction. Then, starting with the instant power for one electron,

$$p = -ev_r(E_{RF,r} + E_{SC,r}) - ev_z E_{RF,z} \,, \tag{17}$$

one may calculate the instant power ΔP_m of all particles contained in the m^{th} segment by adding up their instant powers with corresponding weights. Subtracting ΔP_m from the RF power value $P_{RF,\,m}$ in this segment will give one the RF power value at the entrance to the next segment, i.e.

$$P_{RF,m+1} = P_{RF,m} - \Delta P_m \,. \tag{18}$$

The RF power is related to the normalized wave amplitude as [4]:

$$P_{RF} = \alpha^2 \frac{4\pi\varepsilon_0 m^2 a^2 \omega^5}{e^2 k_z} \Phi, \tag{19}$$

where Φ is the normalized RF power flux. Recalculating the updated normalized wave amplitude from this expression and substituting it into expressions (7-8) would allow taking into account the effects of MP-induced wave attenuation self-consistently.

SUMMARY

We have developed a non-stationary self-consistent quasi-3D Monte-Carlo model for the analysis of multipactor effect in DLA structures. The model includes the effects of axial motion of the particles that were not taken into account in the previous 2D studies. It also includes the effects of wave-particle interaction and correctly calculates the wave attenuation induced by MP. Comparison of the results of this model with experimental data is currently in progress.

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