SPIN DYNAMIC TOOL DEVELOPMENTS AND STUDY REGARDING THE SuperB PROJECT

N. Monseu, J.-M. De Conto, UJF, CNRS-IN2P3, INPG, LPSC, Grenoble, France
F. Méot, BNL, Collider-Accelerator Department, Upton, Long Island, NY, USA
U. Wienands, SLAC, Menlo Park, CA, USA

Abstract

The Zgoubi integrator [1] is a good and universal tool for particle tracking as well as spin tracking [2], and takes into account all machine realistic aspects, like real fields, non-linearities, fringing fields or misalignements. It is used for simulations of the SuperB storage ring. We present the Zgoubi implementation and the methods carried out to estimate the Invariant Spin Field (ISF) evolution of SuperB, on some simple case for validation, and we investigate for some specific polarization behavior.

INTRODUCTION

The proposed SuperB e+/e- asymmetric collider will include a polarized electron Low Energy Ring (LER, 4.18 GeV). The high luminosity foreseen for this collider implies highly non-linear fields, e.g. for Crab Waist scheme, and very strong beam-beam effect[3]. Non-linear and collective effects are difficult to consider in spin design code, mainly due to the large computing time required to take them properly into account. Those effects are at the edge of our numerical capacity. It is known that they strongly affect orbital dynamics, but effects on spin dynamics are not well studied. We follow an approach based on single particle stepwise integration of the Lorentz equation for orbital motion together with the Thomas-BMT equation (1)(without electrical term) for spin motion :

$$\frac{d\vec{S}}{dt} = \frac{e}{\gamma m}\vec{S} \times \left[(1+G\gamma)\vec{B}_{\perp} + (1+G)\vec{B}_{\parallel}\right] \qquad (1)$$

This approach has proved to be efficient for single particle spin dynamics (e.g. [4]), especially because the magnetic field is realistically modeled. This paper proposes to extend this approach to the evaluation of the ISF (or $\vec{n}$, see [5] for its precise definition, $\vec{n}$ on closed orbit is named $\vec{n}_0$). Estimations of polarization degree and rate are foreseen.

Tracking one particle over one thousand turns with Zgoubi is a matter of one minute. Long term tracking for a large set of particles on a single processor is unrealistic. But as Zgoubi performs single particle tracking, parallelisation (one particle for one processor) is possible.

ORBITAL MOTION

Spin dynamics depend on the field encountered by the particles. In this context, it is important, prior to studying spin dynamics, to collect information on orbital dynamics. That was the purpose of [6]. In this part, we remind lattice parameters of the SuperB LER lattice, in its version 12, without any misalignments, including the mismatched spin rotators (Table 1) and present results (Fig. 1 & 2) concerning the orbital parts of simulation performed in the following parts. Tunes are obtained by computing FFT of coordinates, folding the spectra into $[0, 0.5]$. The observation points for every figure of the paper is in the middle of a defocusing quadrupole, located between the last RF cavity and the first bend in the straight section of the ring.

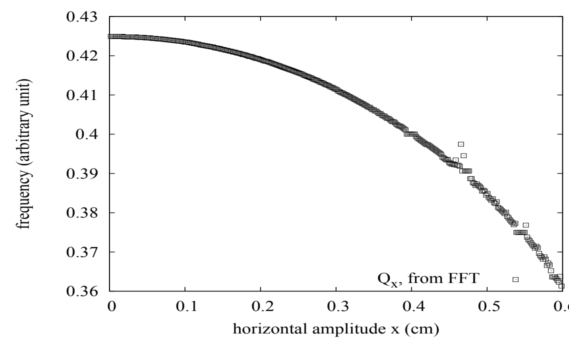

Figure 1: Evolution of the horizontal tune Qx with the amplitude of the horizontal motion.

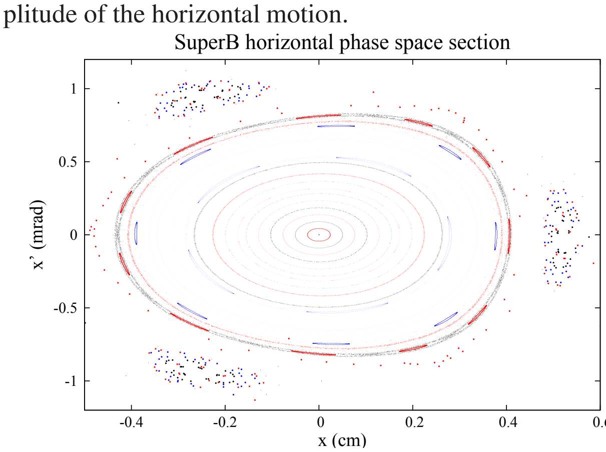

Figure 2: Phase space section of large amplitude ($> 40 \cdot \sigma_x$) horizontal motion in SuperB. Colors are used to distinguish between neighbouring particles.

NUMERICAL METHODS FOR SPIN MOTION

To estimate $\vec{n}_0$ a built-in Zgoubi fitting function ('FIT') is used, with constraint of identical spin coordinates after one turn. For $\vec{n}$ the stroboscopic averaging algorithm is applied on Zgoubi data. This method was developped

Table 1: SuperB LER Lattice Parameters

Energy /GeV	4.18
Orbit length /m	1258.3582
Q_x, Q_y	[42].5749, [18].5949
Q'_x,Q'_y (chromaticities)	-0.624, -0.676
α, $\sqrt{1/\alpha}$	$4.053\,10^{-4}$, 49.67
Max β_x, β_y /m	387.25, 1146.77
β_x, β_y /m (at observation point)	5.5 , 20
ϵ_x, ϵ_y /m	$2.46\,10^{-9}$,$6.15\,10^{-12}$
Max Dx /m	0.5118
ν_{sp}	0.495 ($a \cdot \gamma = 9.48$)
$\vec{n}_0$ (straight section)	(0.0547761, 7.61944e-06, 0.998499)

by Heinemann and Hoffstaetter to get ISF from tracking datas [7]. Tracking a set of particles in a ring over a large enough number of turns, the average of spin components over a small enough phase space volume has no perpendicular components with respect to the local precession axis, if it exists.

We evaluate the spin tune ν_{sp} on closed orbit by measuring the angle between initial spin and spin after one turn. Off closed orbit we use FFT of spin components directly from tracking datas, as in Fig. 3. We can use any of the three spin components, however, the spectrum amplitude at expected precession frequency ($\nu_{sp} = a \cdot \gamma$, with $a = 0.001159$) might be smaller than the amplitude at the vertical tune for some components.

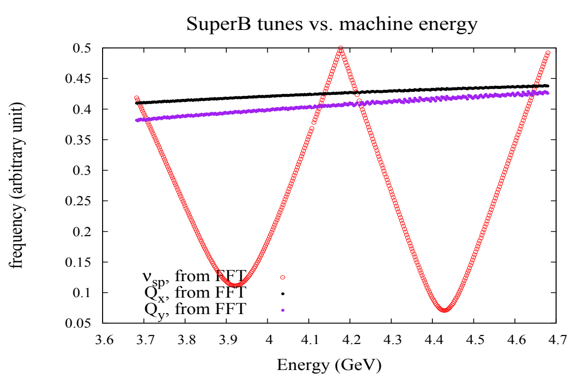

Figure 3: Orbital tunes (Qx and Qy), and spin precession frequency (ν_{sp}), obtained by computing FFT of coordinates, folded in $[0., 0.5]$.

For stroboscopic averaging, we track a large set of particles for many turns, enough for the average to converge : a number of $4 * 10^3$ particles per bin seem to be sufficient to get a precision better than $5 * 10^{-4}$(Fig. 4). We store orbital and spin coordinates at one position in the ring. Then tracking datas are read. A 6D histogram is created by dividing the phase space in small volumes (histogram bins). Each pass of every particle is sorted in a bin. The spin coordinates stored in each bin are averaged. The resulting average is renormalised. The spin field invariance is tested numerically by tracking it for one turn, and comparing the two distributions.

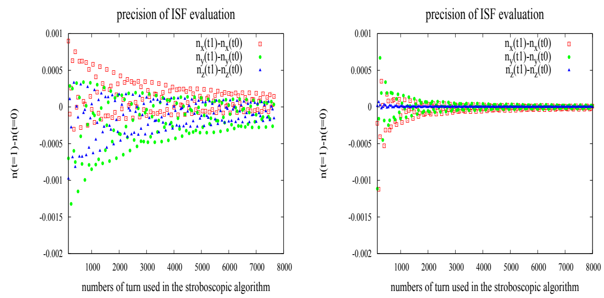

Figure 4: Difference between the components of $\vec{n}_0$ after one turn vs. the number of turns (t) used in the average, for SuperB. Left: at E=3.93GeV, the worst case, as the initial spin was close to vertical. Right : at nominal energy E=4.18GeV.

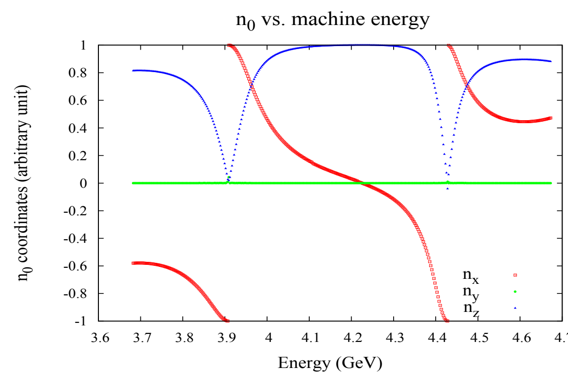

Figure 5: Components of$\vec{n}_0$ in the straight section, obtained by stroboscopic averaging, 10^4 turn/point.

METHOD VALIDATION

To validate the method we compared results on or very close to closed orbits (distance from c.o. less than $10^{-5}cm$), with different methods ('FIT' and stroboscopic averaging), and for different machine lattices. In addition to SuperB LER, comparison with [9] are under progress (not be presented). Figure 5 shows the evolution of the components of $\vec{n}_0$ in the straight section of SuperB LER, as function of the machine energy (all magnets fields are rescaled, together with particle momentum). The motion was only transverse, simulation with synchrotron motion is foreseen. Small discrepancies between the results in [3], sect. 16 (obtained with SLICKTRACK) and ours are probably due to a different LER lattice version. Figure 3 shows orbital tunes that are not constant over the energy range, this points out a problem in magnetic field scaling with energy (in Zgoubi, the fields are used, and one has to precise which magnet to rescale) which is under investigation. .

SPIN MOTION

Our method being validated on the closed orbit case, we studied the effect of amplitude of betatron motion on $\vec{n}$. Figure 6 shows the evolution of the components of ISF with

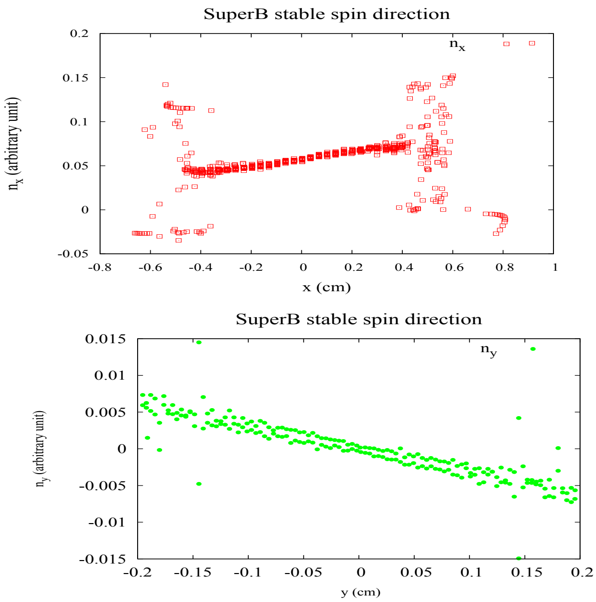

Figure 6: Components of ISF in the straight section, obtained by stroboscopic averaging (preliminary results). Top : n_x components vs. horizontal amplitude, with no initial vertical amplitude. Bottom : n_y vs. vertical amplitude, with no initial horizontal amplitude. In comparison the SuperB beam footprint is $\sigma_x \approx 0.1mm$ and $\sigma_y \approx 11\mu m$ with $\sigma = \sqrt{\epsilon\beta}$. The scattered points at large amplitude are due to a number of particle per bin too small for the average to converge.

the motion amplitude, horizontal and vertical.

Finally, we wanted to test the coherence of the whole : amplitude detuning, spin tune and stroboscopic averaging. If we track particles in a ring rescaled to have a reference energy of $4.13GeV$, the spin tune should be around $\nu_{sp} = a{\cdot}\gamma \approx$ [9].37, close to orbital tunes in Fig. 3. And according to Fig. 1, if the amplitude increase from 0.0 to $0.4\ cm$, the tunes should stand at .57 and increase with amplitude. At an amplitude $x \approx 0.2\ cm$, particles should hit a resonance ($\nu_{sp} + Q_x = integer$), and thus we should see a variation in ISF. In Fig. 7, we plot $\vec{n}_x$ (x components, longitudinal, of $\vec{n}$, calculated with stroboscopic averaging) in function of the amplitude of motion. Indeed $\vec{n}_x$ changes orientation as the resonant values of the tune are approached.

CONCLUSION

We have shown that large amplitude ISF can be computed from Zgoubi tracking datas, and we give a first estimation of the effect of transverse motion on spin dynamics for SuperB. Our approach allows to study any single particle dynamical effect on spin dynamics, and collective effects if they could be realistically simulated with non-linear lenses (i.e. in a 'strong-weak' way). This last point is of great interest for the SuperB project, where, as told in introduction, the beam-beam effect is very strong.

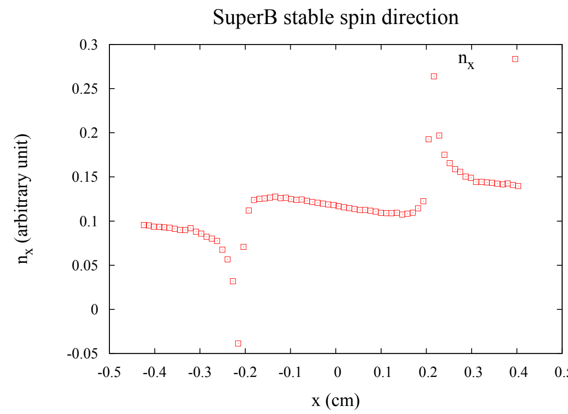

Figure 7: Longitudinal component of invariant spin field ($\vec{n}_x$), obtained by stroboscopic averaging, in function of horizontal amplitude, in the arcs of SuperB LER rescaled at 4.13 GeV. We show the points only for $x' \in [-0.5, 0.5]$, $y = y' = 0$ and with a number of turn used to average bigger than 10^3, thus avoiding the scattered point one could see on Fig. 6.

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