WAKEFIELD CALCULATIONS FOR THE LCLS IN MULTIBUNCH OPERATION*

K. L. F. Bane, SLAC, Menlo Park, CA 94025, USA

INTRODUCTION

Normally the Linac Coherent Light Source (LCLS) operates in single-bunch mode, sending a bunch of up to 250 pC charge at 120 Hz through the linac and the undulator, and the resulting FEL radiation into one of the experimental hutches. With two bunches per rf pulse, each pulse could feed either two experiments or one experiment in a pump-probe type configuration. Two-bunch FEL operation has already been briefly tested at the LCLS, and works reasonably well [1], although not yet routinely.

In this report we study the longitudinal and transverse long-range (bunch-to-bunch) wakefields of the linacs and their effects on LCLS performance in two-bunch mode, which is initially the most likely scenario. The longitudinal wake changes the average energy at the second bunch, and the transverse wake misaligns the second bunch (in transverse phase space) in the presence of e.g. transverse injection jitter or quad misalignments. Finally, we extend the study to consider the LCLS with trains of up to 20 bunches per rf pulse.

In the LCLS the bunch is created in an rf gun, and then passes in sequence through Linac 0, Linac 1, Linac X, Bunch Compressor 1 (BC 1), Linac 2, BC 2, Linac 3, and finally the undulator. In the process the bunch energy reaches 13.5 GeV and peak current 3 kA. In Table 1 we present some machine and beam parameters in three of the linacs that we will use in the calculations: initial beam energy E_0, total accelerator length L, average beta function β_y, bunch peak current I, and rf phase (with respect to crest) ϕ; the final energy of a linac equals E_0 of the following linac, and in Linac 3 is $E_f = 13.5$ GeV. (The X-band linac, with $L = 60$ cm, has wake effects that are small compared to the other linacs, and will not be discussed.) In this report we limit our study to trains of equally populated, equally spaced bunches with a total length of less than 100 ns. The charge of each bunch is $eN_b = 250$ pC.

Table 1: Machine and beam parameters in LCLS linacs.

Linac	E_0 [GeV]	L [m]	β_y[m]	I [A]	ϕ[deg]
0	0.006	6	14	50	0
1	0.135	9	6	50	−20
2	0.25	326	30	250	−32
3	4.2	582	50	3000	0

LONGITUDINAL EFFECTS

The SLAC linac is a disk-loaded, travelling wave structure operating at frequency $f_{01} = 2.856$ GHz, at $2\pi/3$ phase advance per cell. A structure unit is 3 m long and contains 84 cells. The cell period $p = 3.5$ cm, iris thickness $t = 0.6$ cm, average iris radius $a = 1.2$ cm, and average cavity radius $b = 4$ cm. The structure is constant gradient, and a becomes smaller as one moves from beginning to end of the structure; b changes in the opposite direction in order to keep f_{01} unchanged. As a result the first band dipole modes of a structure are approximately linearly detuned.

The longitudinal (monopole) wake excited by a bunch can be written as

$$W_z(t) = \sum 2\kappa_{0n} \cos(2\pi f_{0n} t) e^{-\pi f_{0n} t/Q_{0n}} \ , \qquad (1)$$

with t time after the bunch, and f_{0n}, κ_{0n}, Q_{0n}, the longitudinal mode frequencies, loss factors, and quality factors.

The longitudinal interaction is dominated by the fundamental mode wake. Since the bunches are spaced by h/f_{01} with harmonic number h an integer, the fundamental interaction will be a linear bunch-to-bunch energy droop. (Since $Q_0 \sim 10,000$—the $1/e$ time ~ 1 μs—we can neglect the damping factor.) The voltage at bunch m, at the end of a linac, is given by $V_w = -2eN_bL\kappa_{01}m$; the fundamental mode loss factor $\kappa_{01} = 20$ V/pC/m in the SLAC linac. With multiple bunches, to compensate the wake effect a linear chirp in the rf amplitude will need to be introduced.

In the case of two bunches, the relative energy difference of the bunches due to the wake is δ_w =(4.4, 3.6, 7.8, 4.3)$\times10^{-4}$ at the end of Linac (0, 1, 2, 3). We can compare this to the rf amplitude and phase jitter in the LCLS linacs, which are approximately (3.5–10)$\times10^{-4}$ and 0.05 deg, respectively [2]. The voltage seen by a bunch is the vector sum of the applied rf (V_{rf} at phase ϕ) plus the wake (V_w at phase pi); thus the wake effect is equivalent to an rf amplitude and phase change of $\delta V_{rf} = -$(4.7, 6.9, 5.9, 6.3) $\times10^{-4}$ and $\Delta\phi = -$(0, 0.014, 0.021, 0) degrees in Linac (0, 1, 2, 3). Thus, for two bunches and under normal operating conditions, the wake effect will barely be noticeable above the rf jitter.

To estimate the contribution of the higher longitudinal modes, we have performed time domain calculations on a 10-cell model of the SLAC linac structure using the 2D code ECHO [3]. (If needed, the equivalent 3D calculation can be performed to find the HOM effect more precisely.) The driving bunch length in the simulations was $\sigma_z = 2$ mm, and the wake was calculated to 50 ns behind the bunch. The simulations do not include the rf couplers (that can couple out HOMs), and thus give a pessimistic, upper limit of the wake effect. By taking the Fourier transform of the resulting wake, we find that there are 5 dominant HOM's in the range [5.9, 12.3] GHz. The beating of the modes leads to a wake variation—at the fundamental mode spacing—of $\sim$ 30% rms.

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TRANSVERSE EFFECTS

Double-Band Wake Calculation

The modes of the first two dipole bands have been calculated using a double-band circuit model [4].[1] For the 84-cell SLAC structure the program obtains 168 mode frequencies f_{1n} and kick factors κ_{y1n} (see Fig. 1). The first half of the modes, those in the first band with frequencies in the range [4.14, 4.35] GHz, dominate. The second band modes are needed in the calculation to give the correct spacing of the first band modes. The first mode, with $f_{11} = 4.140$ GHz, $\kappa_{y11} = 82$ V/(nC mm m), is dominant. When the accelerator was built [6] the first five mode frequencies and the first mode kick factor were measured and calculated. Our results agree well with those: *e.g.* the measured $f_{11} = 4.13964$ GHz, $\kappa_{y11} = 77$ V/(nC mm m).

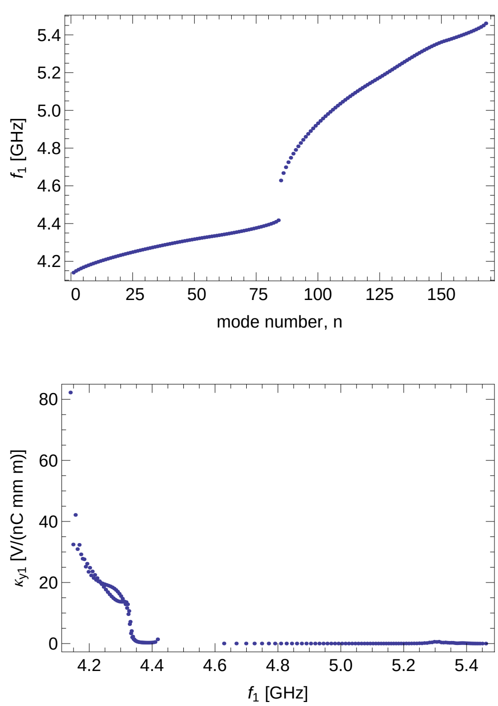

Figure 1: Dipole mode frequencies and kick factors for the SLAC structure, as obtained by the two-band circuit model.

The transverse (dipole) wake excited by a bunch is

$$W_y(t) = \sum 2\kappa_{y1n} \sin(2\pi f_{1n} t) e^{-\pi f_{1n} t/Q_{1n}} \ . \qquad (2)$$

In order to weaken the first mode coherence in the SLAC linac, 1/3$^{\rm rd}$ of the structures were dimpled to increase the frequency by $+2$ MHz, and another 1/3$^{\rm rd}$ by $+4$ MHz. Note that neither this modification nor the finite Q ($Q_{1n} \sim 18,000$) have much effect in the first 100 ns of the wake.

The wake is shown in Fig. 2 over long and short time ranges. The wake envelope begins by decaying as a sinc function, since $\kappa_{y1n} dn/df$ (not shown; with dn/df the density of modes) is a flat-top of width 200 MHz, resulting in the first zero at $(200\ \mathrm{MHz})^{-1} = 5$ ns. After 100 ns the wake partially recoheres. In the lower plot the red dots give the wake at spacings h/f_{01} behind the driving charge, where a second bunch could be. We see primarily the beating of modes 01 and 11; the maximum interaction is then when $f_{11}h \approx f_{01}(n_1 \pm \frac{1}{4})$, where n_1 is a positive integer, which occurs at a spacing of $h = 6$ fundamental periods. From the plot we see that the maximum actually occurs for $h = 6$ and 7.

[1] This calculation was performed before, in Ref. [5], though here we give more details of the results.

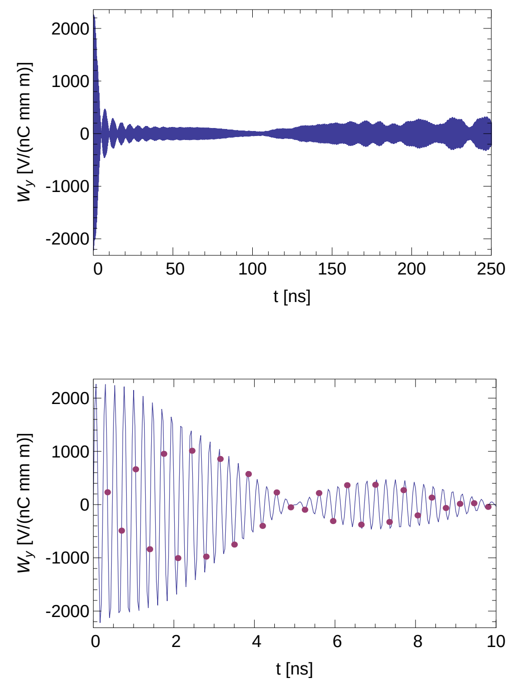

Figure 2: Long-range dipole wake of SLAC linac structure over two time ranges, with the bucket locations indicated by the red dots (lower figure).

Beam Break-Up

For an equally populated and equally spaced train of bunches, with charge per bunch eN_b and spacing h/f_{01}, the equations of motion are given by

$$\frac{1}{E(s)}\frac{d}{ds}\left[E(s)\frac{dy_m(s)}{ds}\right] + \frac{y_m(s)}{\beta_y^2(s)} = \frac{e^2 N_b}{E(s)} \sum_{j=1}^{m-1} y_j(s) W_y([m-j]h/f_{01}) \ , \qquad (3)$$

where $y_m(s)$ is offset of bunch m, and s is position along the machine. The bunches are approximated as point particles, and the focusing as being smooth. With M bunches, Eq. 3 represents M coupled equations that, given initial conditions, we solve numerically using Mathematica.

To simulate the effect of injection jitter into a linac we begin with all bunches offset by a unit amount. We define a wake sensitivity parameter $\xi = \max(\|r_m - r_1\|/\|r_1\|)$, with r_m the offset, in transverse phase space, of the $m^{\rm th}$ bunch at the end of a linac (Fig. 3 sketches the situation for the case of 2 bunches).

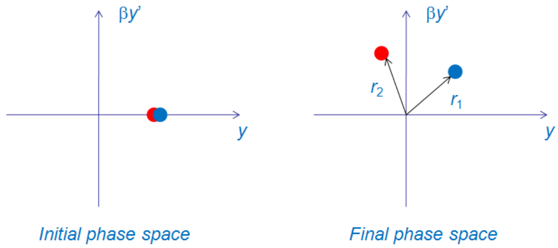

Figure 3: Sketch to explain the wake sensitivity parameter in the case of two bunches. Bunch 1 is leading.

Chao et al solved this problem analytically for the *single bunch* instability using a perturbation approach [7]; their result was given in terms of a strength parameter Υ. Similarly, for the multi-bunch effect the result can also be characterized by a strength parameter, which is given (in the case of constant β_y; assuming adiabatic acceleration, $\beta_y(E_f - E_0)/E_0 L \ll 1$) by [8]

$$\Upsilon_m = \frac{e^2 N_b L S_m \beta_y}{2(E_f - E_0)} \ln\left(\frac{E_f}{E_0}\right) , \tag{4}$$

with the sum wake $S_m = \sum_{i=1}^{m} W_y([i-1]h/f_{01})$. For the case of two bunches and no acceleration: $y_1 \sim \cos(s/\beta_y)$, $(y_2 - y_1) \sim \Upsilon_2 (s/L) \sin(s/\beta_y)$, and $\xi = \Upsilon_2$.

RESULTS

Two Bunches

In Fig. 4 we plot the ξ *vs* h obtained numerically for the case of two bunches (blue circles), as well as Υ_2 (the red curve). We see good agreement. We see the maximum sensitivity for $h = 6$, 7, as expected. If the acceptable offset of the beam from the design orbit is say $0.1\sigma_y$ (for FEL considerations), then the result means that in the worst situation ($h = 6$ or 7) the second bunch will be offset at the end of Linac 2, with respect to the first bunch, by $\sim 0.1\sigma_y$. Note that in the linear regime, the wake kick is in quadrature to the oscillation. Thus the tolerance to injection jitter will be reduced by the factor $(1 + \xi^2)^{-1/2}$; for $h = 6$ or 7 it is reduced by the factor 0.7.

The other linacs in the LCLS yield very much the same pattern in the numerically obtained ξ *vs* h, though the effect is weaker, with the amplitude of ξ in the ratio (0.16, 0.05, 1, 0.46) for Linac (0, 1, 2, 3). Linac 2 is most sensitive.

Twenty Bunches

We've solved Eqs. 3 in Linac 2 for up to $M = 20$ bunches and harmonic number up to $h = 15$. A plot representing the results $\xi(h, M)$ is shown in Fig. 5. The plot is color coded in unit steps, where color i represents values $i - \frac{1}{2} < \xi < i + \frac{1}{2}$. From the way ξ is defined, for any h the

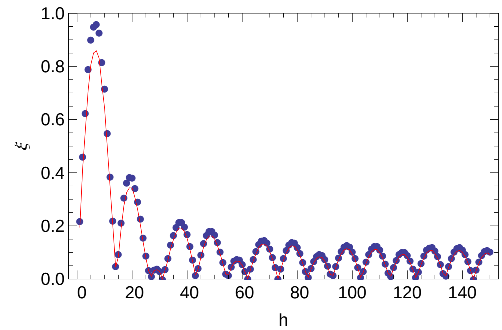

Figure 4: The case of two bunches: ξ *vs* h in Linac 2 (blue circles). Υ_2 is shown in red.

curve monotonically increases as M increases. We see that the most sensitive value of h is 2: for $M \gtrsim 10$, $\xi \sim 10$, and the injection jitter tolerance is $(\frac{1}{10})^{\rm th}$ that of the no-wake case. For $h \geq 14$, $\xi \lesssim \frac{1}{2}$; at $h = 14$ and $M = 20$, $\xi = 0.1$. For many bunches, $\max(\Upsilon_m)$ is still a useful strength indicator in that it correlates with ξ, though it no longer equals ξ; for $M = 20$: with $h = 2$ (14) it equals 3.8 (0.25). In conclusion, we see that multiple bunches in the LCLS will tend to induce a significant wake effect, unless the bunch spacing is kept to $\gtrsim 14$ buckets.

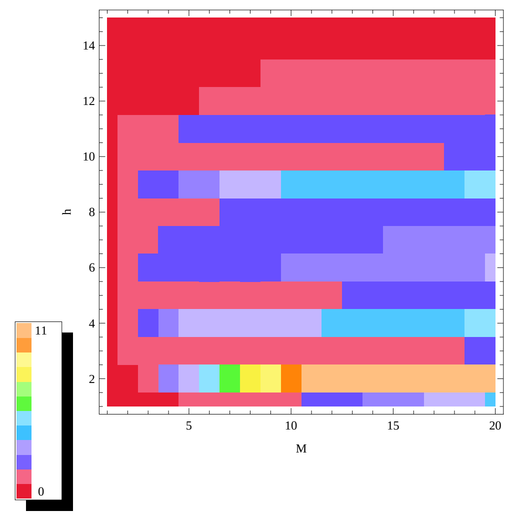

Figure 5: Multiple bunches in Linac 2: injection sensitivity ξ *vs* harmonic number h and number of bunches M.

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