IMPEDANCE CALCULATIONS FOR SIMPLE MODELS OF KICKERS IN THE NON-ULTRARELATIVISTIC REGIME

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Abstract

Kicker magnets are usually significant contributors to the beam coupling impedance of particle accelerators. An accurate understanding of their impedance is required in order to correctly assess the machine intensity limitations. The field matching method derived by H. Tsutsui for the longitudinal and transverse dipolar (driving) impedance of simple models of kickers in the ultrarelativistic regime was already extended to the non-ultrarelativistic case, and to the quadrupolar (detuning) impedance in the ultrarelativistic case. This contribution presents the extension to the quadrupolar impedance in the non-ultrarelativistic case, as well as benchmarks with other available methods to compute the impedance. In particular, all the components of the impedances are benchmarked with (1) Tsutsui's model, i.e. in the ultrarelativistic limit, (2) the model for the flat chamber impedance recently computed by N. Mounet and E. Métral, in the case of finite relativistic gamma, and with (3) CST Particle Studio simulations.

INTRODUCTION

Kickers can be major contributors to the total beam coupling impedance and cause of heating issues in particle accelerators [1]. Previous studies were done in order to compute the impedance related to these devices. B. Zotter and E. Métral computed the impedance of a flat chamber [2] using Yokoya factors [3], valid for $\beta = 1$ and good conductor boundaries, but, in order to take into account the specific quadrupolar part of the impedance of kickers made of dispersive material, it turned to be necessary to use Tsutsui's model for the longitudinal [4], transverse driving [5] and detuning impedances [6]. In the frame of this model, Tsutsui's model was recently extended for machines operating in the non-ultrarelativistic regime (i.e. $\beta < 1$) as the case of the Rapid Cycling Synchrotron ring (RCS) in the China Spallation Neutron Source (CSNS) [7]. In this contribution this latter theory is extended to the calculation of the quadrupolar transverse impedances.

FIELD MATCHING METHOD

The Field Matching Method is commonly used to deal with discontinuities and multi-layer problems. In our case we will consider the geometry reported in Fig. 1.

The beam is a point-like charge travelling at the centre of the kicker exciting an electromagnetic (e.m.) field both in vacuum and in ferrite.

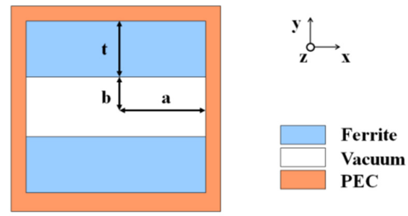

Figure 1: Kicker simplified geometry. The structure is infinitely long in the z direction.

To compute this field, we could solve the Maxwell equations in presence of the source current (the beam) but it turns out to be difficult due to singularity problems. In order to overcome this limitation a primary field is computed from the source beam, the most simple as possible. This primary field plus an unknown scattered field both in vacuum and ferrite is then matched at the separation between layers. This will compensate the primary field satisfying the boundary conditions. Eventually, in order to compute the impedance we need to:

- Set the correct source beam and its primary field.
- Divide the geometry in sub-domains in which the scattered e.m. fields are calculated.
- Match the e.m. fields at the boundaries of each domain.
- Compute the impedance.

Primary Fields

In order to compute the quadrupolar impedance, a single point-like source current placed at the geometrical centre of the kicker moving with velocity $v = \beta c$ in vacuum can be considered. As primary fields $X^{(S)}$ we will consider the e.m. fields generated by the charge in free space (as written above, this is not a solution of our problem but a "first guess" that will be compensated by the scattered field in order to satisfy the boundary conditions). Solving Maxwell equations [8] leads us to:

$$
\begin{aligned}
E_x^{(S)}(x,y) &= \frac{Z_0 q}{2\pi\beta} K_1(k_r r) k_r \frac{x}{\sqrt{x^2+y^2}}\,, \\
E_y^{(S)}(x,y) &= \frac{Z_0 q}{2\pi\beta} K_1(k_r r) k_r \frac{y}{\sqrt{x^2+y^2}}, \\
E_z^{(S)}(x,y) &= j\frac{Z_0 q}{2\pi\beta\gamma} k_r K_0(k_r r)\,, \\
H_x^{(S)}(x,y) &= -\frac{\beta}{Z_0} E_y^{(S)}(x,y), \\
H_y^{(S)}(x,y) &= \frac{\beta}{Z_0} E_x^{(S)}(x,y)\,.
\end{aligned}
\tag{1}
$$

where $k_r = \omega/\gamma v$, $\gamma = 1/\sqrt{1-\beta^2}$ is the relativistic factor, $Z_0 \cong 377\,\Omega$ the characteristic vacuum impedance, $q \cong 1.602 \times 10^{-19}$ C the proton charge, $K_{0,1}$ respectively the modified Bessel function of order 0 and 1. Taking into account the parallel PEC plates at $x = \pm a$, i.e. adding the image currents, one gets:

$$
\begin{aligned}
E_x^{(S)} &= \frac{Z_0 q}{2\pi\beta} k_r \sum_{m=-\infty}^{+\infty} (-1)^m \frac{x-2ma}{r_m} K_1(k_r r_m),\\
E_y^{(S)} &= \frac{Z_0 q}{2\pi\beta} k_r \sum_{m=-\infty}^{+\infty} (-1)^m \frac{y}{r_m} K_1(k_r r_m),\\
E_z^{(S)} &= j\frac{Z_0 q}{2\pi\beta\gamma} \sum_{m=-\infty}^{+\infty} (-1)^m \frac{y}{r_m} K_0(k_r r_m),\\
H_x^{(S)} &= -\frac{q}{2\pi} k_r \sum_{m=-\infty}^{+\infty} (-1)^m \frac{y}{r_m} K_1(k_r r_m),\\
H_y^{(S)} &= \frac{q}{2\pi} k_r \sum_{m=-\infty}^{+\infty} (-1)^m \frac{x-2ma}{r_m} K_1(k_r r_m).
\end{aligned}
\tag{2}
$$

where $r_m = \sqrt{(x-2ma)^2 + y^2}$.

Scattered Fields

In vacuum, the e.m. fields $X^{(v)}$ are solution of the Helmholtz equation taking into account the PEC boundary conditions at $x = \pm a$. We have:

$$
\begin{aligned}
E_z^{(v)} &= \sum_n A_n cos(k_{1n}x) cos(k_{2n}y),\\
H_z^{(v)} &= \sum_n B_n sin(k_{1n}x) sin(k_{2n}y),\\
E_x^{(v)} &= \sum_n \frac{(\omega\mu_0 k_{2n} B_n - \mathrm{k}\mathrm{k}_{1n}\mathrm{A}_n)}{-jk_r{}^2} sin(k_{1n}x) cos(k_{2n}y),\\
E_y^{(v)} &= \sum_n \frac{(\omega\mu_0 k_{1n} B_n + \mathrm{k}\mathrm{k}_{2n}\mathrm{A}_n)}{jk_r{}^2} cos(k_{1n}x) sin(k_{2n}y),\\
H_x^{(v)} &= \sum_n \frac{(kk_{1n} B_n + \omega\epsilon_0 \mathrm{k}_{2n}\mathrm{A}_n)}{-jk_r{}^2} cos(k_{1n}x) sin(k_{2n}y),\\
H_y^{(v)} &= \sum_n \frac{(kk_{2n} B_n - \omega\epsilon_0 \mathrm{k}_{1n}\mathrm{A}_n)}{-jk_r{}^2} sin(k_{1n}x) cos(k_{2n}y),
\end{aligned}
\tag{3}
$$

where $k = \omega/v$. The eigenvalues are $k_{1n} = (2n+1)\pi/2a$ with $\in (0, +\infty)$. The k_{2n} are instead constrained only by the separability condition $k_{1n}{}^2 + k_{2n}{}^2 = -k_r{}^2$.

In ferrite the e.m. fields $X^{(f)}$ have analogous expression. Imposing the PEC surfaces at $y = \pm(b+t) = \pm d$ we get:

$$
\begin{aligned}
E_z^{(f)} &= \sum_n C_n \cos(g_{1n}x) sin(g_{2n}\tilde{y}),\\
H_z^{(f)} &= \sum_n D_n sin(g_{1n}x) cos(g_{2n}\tilde{y}),\\
E_x^{(f)} &= \sum_n \frac{(\omega\mu g_{2n} D_n + \mathrm{k}\mathrm{g}_{1n}\mathrm{C}_n)}{j(k^2 - k_0^2\mu_r\varepsilon_r)} sin(g_{1n}x) sin(g_{2n}\tilde{y}),\\
E_y^{(f)} &= \sum_n \frac{(\omega\mu g_{1n} D_n - \mathrm{k}\mathrm{g}_{2n}\mathrm{C}_n)}{j(k^2 - k_0^2\mu_r\varepsilon_r)} cos(g_{1n}x) cos(g_{2n}\tilde{y}),\\
H_x^{(f)} &= \sum_n \frac{(kg_{1n} D_n - \omega\varepsilon\mathrm{g}_{2n}\mathrm{C}_n)}{-j(k^2 - k_0^2\mu_r\varepsilon_r)} cos(g_{1n}x) cos(g_{2n}\tilde{y}),\\
H_y^{(f)} &= \sum_n \frac{(kg_{2n} D_n + \omega\varepsilon\mathrm{g}_{1n}\mathrm{C}_n)}{j(k^2 - k_0^2\mu_r\varepsilon_r)} sin(g_{1n}x) sin(g_{2n}\tilde{y}),
\end{aligned}
\tag{4}
$$

where $\tilde{y} = y - d$ and $k_0 = \omega/c$. The eigenvalues are $g_{1n} = (2n+1)\pi/2a$. The g_{2n} are instead constrained only by the separability condition $g_{1n}{}^2 + g_{2n}{}^2 = k_0^2\mu_r\varepsilon_r - k^2$. The material characteristics are plugged in the permeability $\mu = \mu_0\mu_r$ and permittivity $\varepsilon = \varepsilon_0\varepsilon_r$.

Matching

Once the e.m. fields are derived we can apply the continuity relations [9] on the separation layers vacuum-ferrite at $y = \pm b$. We have a set of 4 x-dependent equations in 4 n-vector unknowns A_n, B_n, C_n, D_n.

I)	$E_z^{(S)} + E_z^{(v)} = E_z^{(f)}$	II)	$H_z^{(S)} + H_z^{(v)} = H_z^{(f)}$
III)	$E_x^{(S)} + E_x^{(v)} = E_x^{(f)}$	IV)	$E_y^{(S)} + E_y^{(v)} = \varepsilon_r E_y^{(f)}$

The system can be solved expanding the source term in Fourier series and matching each modal component of the fields. The full expressions of the coefficients were derived in [7].

Quadrupolar Impedance Calculation

To get the transverse quadrupolar (or detuning) impedance we have to displace a test particle with respect to the beam trajectory on (x, y)=(0, 0) by a quantity ξ in the plane of interest [10].
The impedance is given by:

$$
Z_x^{det} = \frac{j}{P}\int_{-\infty}^{+\infty} [E_x(\xi,0) - v\mu\, H_y(\xi,0)] e^{jkz} dz,
$$
$$
Z_y^{det} = \frac{j}{P}\int_{-\infty}^{+\infty} [E_y(0,\xi) - v\mu\, H_x(0,\xi)] e^{jkz} dz,
$$

where ξ is the displacement of the test particle with respect to the source beam trajectory and $P = q\xi$ is the dipole moment. Since $\nabla \times E = -j\omega\mu H$ we get:

$$
\frac{Z_x^{det}}{L} = -\frac{1}{kP}\frac{\partial E_z(x,0)}{\partial x}, \qquad \frac{Z_y^{det}}{L} = -\frac{1}{kP}\frac{\partial E_z(0,y)}{\partial y}.
$$

All these formulas have to be evaluated respectively in $x = \xi$ and $y = \xi$. Substituting the field E_z in vacuum from the first equation in Eq. (3) we eventually obtain the formula for the detuning impedance:

$$
\frac{Z_x^{det}}{L} = \frac{1}{kq}\sum_n k_{1n}^2 A_n, \qquad \frac{Z_y^{det}}{L} = \frac{1}{kq}\sum_n k_{2n}^2 A_n.
$$

It is worth to notice that the two components do not cancel each other if $\gamma < \infty$, even if their difference could be negligible (see also [11 Eq. (22)] and Fig. 2 in the next section).

APPLICATIONS AND BENCHMARKS

The current model was already applied in the past to compute longitudinal and dipolar impedances: the non-ultrarelativistic formulae obtained in [7] were compared with the ultrarelativistic case analysed by Tsutsui. In Fig. 2 we present a comparison for the quadrupolar impedance of the MKE.61651 SPS kicker with various relativistic β, showing the expected convergence to the ultra-relativistic model for β = 1.

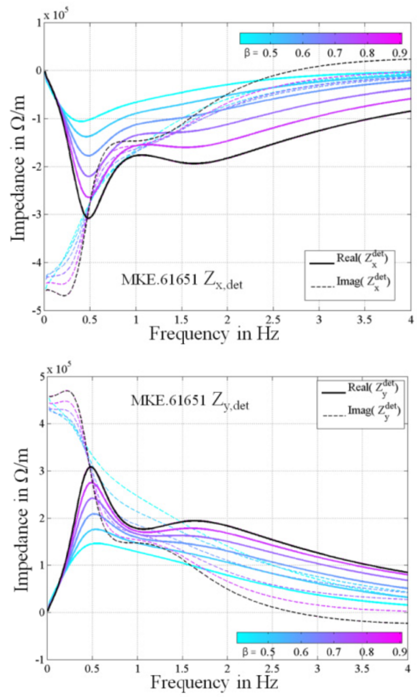

Figure 2: Convergence of the horizontal (top) and vertical (bottom) quadrupolar component to the ultra-relativistic model. The coloured real part (full thick lines) and imaginary part (thin dashed line) are given for different values of β, converging to the black lines given by the ultra-relativistic Tsutsui's model.

The model was successfully cross-checked with the flat chamber model developed by N. Mounet and E. Métral [11] pushing the half width "*a*" to infinity (see Fig. 3).

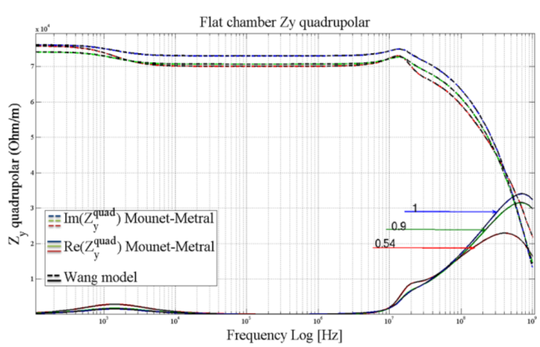

Figure 3: Mounet/Métral - Wang model [7] vertical quadrupolar impedance comparison for β values of 0.54, 0.9 and β ≈ 1 for MKE.61651. Real part in thick lines, imaginary part in dashed thin lines.

Another comparison was done with CST Particle Studio which recently supports open boundaries with β < 1 (Fig. 4). The agreement in terms of quadrupolar wake potential is good but further cross-checks are foreseen in order to explain the little discrepancy.

After these successful benchmarks, we show in Fig. 5 the transverse impedances computed at injection for the different CERN machines: PSB (β ≈ 0.34), PS (β ≈ 0.91), SPS and LHC (β ≈ 1).

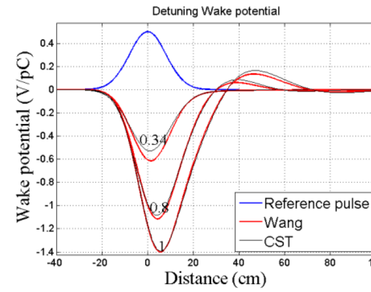

Figure 4: CST – Wang model [7] transverse quadrupolar wake potential comparison for SPS kicker MKE.61651 for β values 0.34, 0.8 and 1.

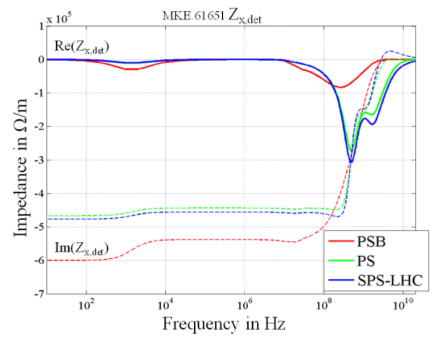

Figure 5: Detuning horizontal impedance for CERN PSB, PS, SPS-LHC machines for SPS kicker MKE.61651.

These plots reveal a decrease in the imaginary part of the impedance and a local increase of the real part at high frequencies with β.

CONCLUSION

The impedance for a 2D simple kicker model was successfully extended and benchmarked with theory and numerical simulations for all the components of the beam coupling impedance in the non ultrarelativistic case. Further extension to the 3D model is foreseen.

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