3D BEAM DYNAMIC SIMULATION IN HEAVY ION SUPERCONDUCTING DRIFT TUBE LINAC

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Abstract

The superconducting (SC) ion linac conventionally consists of some different classes of the identical cavities. Each cavity is based on a SC structure with a high accelerating gradient. The low charge state beams require stronger transverse focusing. This focusing can be reached with the help of SC solenoid lenses. In this paper beam dynamics simulation is done by smooth approximation and full field. Traditionally only the Coulomb field takes into account for low energy beams. In this paper the computer simulation of heavy ion beam dynamics in superconducting linac will carry out by means of the "particle-in-cell" method. Simulation results will present.

INTRODUCTION

High-current accelerators have great perspectives for problems of thermonuclear fusion, safe nuclear reactors, transmutation of radioactive wastes and free electron lasers. A large number of low energy particle accelerators are applied in micro- and nanoelectronics, material science, including the study of new construction materials for nuclear industry, in medical physics, in particular for cancer treatment by using of the accelerators of protons and light ions, in radiation technology. It is proposed to use one universal accelerator, consisting of independently phased cavities and solenoids sequence to solve these problems.

During the past 40 years some different types of short superconducting low and medium energy cavities have been developed for ion and proton acceleration. Linacs are based on niobium SC interdigital cavities, which can provide an accelerating gradient per cavity about 1 MV. Such structures can be used for ion acceleration with different mass-charge ratio in the low energy region [1] and for proton linac in the high-energy region (SNS, JHF, ESS project). Ions are accelerated and slipping relative to the RF wave in dependence of the ratio between the particle velocity β and the phase velocity of the wave β_G. The beam can be both longitudinally stable and accelerated in the whole system [2] by control the driven phase of the accelerating structure and the distance between the cavities. Together with the higher accelerating rate in SC linac the defocusing factor is much higher in comparison to the normal conducting linear accelerator. The beam focusing can be provided by SC solenoids which follow each the cavity [1]. The conditions of longitudinal and transverse beam stability for the structure consisting from the periodic sequence of cavities and solenoids were studied early using transfer matrix calculation [3]. It is very important to know the bucket size since it relates to the longitudinal RF focusing in SC linac design. The smooth approximation can be used in order to investigate the nonlinear ion beam dynamics in such accelerating structures and to calculate the longitudinal and transverse acceptances [4,5].

The accurate treatments of the beam own field and its influence to the beam dynamics is one of the main problems for developers of high-current RF accelerators. Coulomb field, radiation and beam loading effect are the main factors of the own space charge. Typically, only one of the components is taken into account for different types of accelerators. It is Coulomb field for low energy linacs. That is why threedimensional self-consistent computer simulation of high current beam bunching with transverse and longitudinal motion coupling is very actual.

The most useful methods for self-consistent problem solving are the method of kinetic equation and the method of large particles. This problem can be solved by means of the well-known large particles methods as particle in cell (PIC) or cloud in cell (CIC). There is no easy method for the dynamics simulation that takes into account the beam loading effect.

The purpose of the present work is self-consistent high current beam dynamics investigation in accelerating structures by means of three-dimensional program BEAMDULAC-SCL. The BEAMDULAC code is developing in MEPhI since 1999 for high current beam dynamics simulation in linear accelerators and transport channels. The Runge-Kutta 4th order method is used for the integration of differential equations of motions. The algorithm of BEAMDULAC-SCL code uses any previously defined initial particles distribution in 6D phase space to calculate the Coulomb field distribution. As a result, the new coordinates, velocities and phases of large particles are determined, and the new values of the selfconsistent field is defined. The traditional CIC method is used for Coulomb field calculation.

BASIC RELATIONS

As was shown early [1] the beam focusing in such SC linac can be realized by the solenoid field near $B \sim 20$ T. The value of magnetic field B can be reduced by using of advanced phase focusing addition (APF). We can achieve the acceleration and the focusing by less magnitude of magnetic field B by adjusting the drive phase (φ_1 and φ_2) of the two cavities [2]. Adding a focusing solenoid into focusing period will also allow separate control of the transverse and longitudinal beam dynamics. The analysis of the accelerator parameters the and the stability of beam motion is carrying out to the two-dimensional case. The

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general axisymmetric equations of ion motion moving inside an accelerator can be written as

$$\begin{aligned}\frac{d}{dt}\left(\gamma\frac{dz}{dt}\right)&=\frac{eZ}{Am}E_z(\vec{r},t)-\frac{e^2Z^2}{2A^2m^2\gamma}\frac{\partial}{\partial z}A_\varphi^2,\\ \frac{d}{dt}\left(\gamma\frac{dr}{dt}\right)&=\frac{eZ}{Am}E_r(\vec{r},t)(1-\beta\beta_G)-\frac{e^2Z^2}{2A^2m^2\gamma}\frac{\partial}{\partial r}A_\varphi^2.\end{aligned}\qquad(1)$$

In every cavity the acceleration RF field of periodic H-cavity can be represented as an expansion of spatial harmonics

$$\left.\begin{aligned}E_z&=E_0\sum \mathrm{I}_0(h_n r)\cos(h_n(z-z_i))\cos(\omega t),\\ E_r&=E_0\sum \mathrm{I}_1(h_n r)\sin(h_n(z-z_i))\cos(\omega t),\end{aligned}\right\}\qquad(2)$$

where E_0 is the amplitude of RF field at the axis ($E_0\neq 0$ if $-L_r/2<z-z_i<L_r/2$), $h_n=\pi/D+2\pi n/D$, $n=0, 1, 2, \ldots$, $D_i=\beta_G\lambda/2$ is the period length of the cavity, L_r is the cavity length, z_i is the coordinate of the i-th cavity center. I_0, I_1 are modified Bessel function. In our case the reference particle velocity β_c and the geometrical velocity β_G are closely in each class of the identical cavities. Retaining in (2) only zeroth harmonic we can use the traveling wave system. In this system ωt can be replaced by $h_0(z-z_i)+\varphi_{0i}$, where φ_{0i} is the RF phase when the reference particle traverses the cavity center. In equation (1) the value A_φ is the azimuthal vector-potential of the magnetic field in every solenoid ($\boldsymbol{B}=\mathrm{rot}\boldsymbol{A}$).

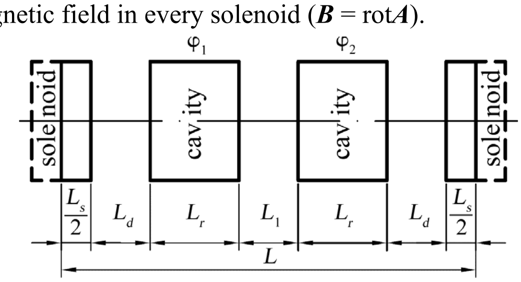

Figure 1: Layout of structure period.

BEAM DYNAMICS IN SMOOTH APPROXIMATION

Let us consider the particle acceleration in the polyharmonic fields of the cavities and solenoids. The ion dynamics in such periodic structure is complicated. The particles trajectories can be presented as a sum of the slowly term and a fast oscillation term with a period L. The normalized particle velocity deviation with respect to the reference particle velocity, $\Delta\beta$, can be represented as a sum of a slow motion term and a fast oscillation term too.

Following Ref. [5] one can apply the averaging over the fast oscillations to obtain the phase (ψ) and radial (ρ) motion equations in the smooth approximation.

$$\begin{aligned}&\frac{d^2\psi}{d\xi^2}+3\left[\frac{d}{d\xi}(\ln\beta\gamma)\right]\frac{d\psi}{d\xi}=-\frac{\partial\overline{U}_{eff}}{\partial\psi},\\ &\frac{d^2\rho}{d\xi^2}+\left[\frac{d}{d\xi}(\ln\beta\gamma)\right]\frac{d\rho}{d\xi}=-\frac{\partial\overline{U}_{eff}}{\partial\rho},\end{aligned}\qquad(3)$$

where $U_{eff}=U_0+U_1+U_2$ is the effective potential function and

$$U_0=-4\alpha[\psi\{\cos\varphi_1+\cos\varphi_2\}-\mathrm{I}_0(\rho)\{\sin(\varphi_1+\psi)+\sin(\varphi_2+\psi)\}]$$

$$\begin{aligned}U_1=\chi_1\alpha^2\{&2(\cos\varphi_1+\cos\varphi_2)(\sin\varphi_1+\sin\varphi_2)\psi+\\ &+\mathrm{I}_0^2(\rho)[\cos(\varphi_1+\psi)+\cos(\varphi_2+\psi)]^2+\\ &+\mathrm{I}_1^2(\rho)[\sin(\varphi_1+\psi)+\sin(\varphi_2+\psi)]^2\}+\\ +\chi_2\alpha^2\{&2(\cos\varphi_1-\cos\varphi_2)(\sin\varphi_1-\sin\varphi_2)\psi+\\ &+\mathrm{I}_0^2(\rho)[\cos(\varphi_1+\psi)-\cos(\varphi_2+\psi)]^2+\\ &+\mathrm{I}_1^2(\rho)[\sin(\varphi_1+\psi)-\sin(\varphi_2+\psi)]^2\},\end{aligned}\qquad(4)$$

$$\begin{aligned}U_2=-\frac{\chi_3}{2}\alpha\tilde{B}\rho\mathrm{I}_1(\rho)\frac{L_{sol}}{L}[\sin(\varphi_1+\psi)+\sin(\varphi_2+\psi)]+\\ \chi_4\tilde{B}^2\frac{L_{sol}^2}{L^2}\rho^2++\tilde{B}\frac{L_{sol}}{L}\frac{\rho^2}{2}.\end{aligned}$$

Here $\alpha=\dfrac{\pi eZUL}{2A\lambda mc^2\beta_g^3\gamma_g^3}$ is the interaction parameter, $\tilde{B}=(eZBL/2Amc\beta_c\gamma_c)^2$ is the focusing coefficient.

Table 1: χ-values for different L_r/L

L_r/L	χ_1	χ_2	χ_3	χ_4
0	1/6	1/2	2/3	1/24
1/4	1/24	1/3	3/8	3/128
1/2	0	1/6	1/6	1/96

We take into account the coherent oscillations of bunches and the effective potential function describe slowly oscillations in the reference particle frame in this expression for U_{eff}.

The analysis of the effective potential function makes possible the study the condition at which the phase and radial stability of the beam is achieved and calculate the longitudinal acceptance.

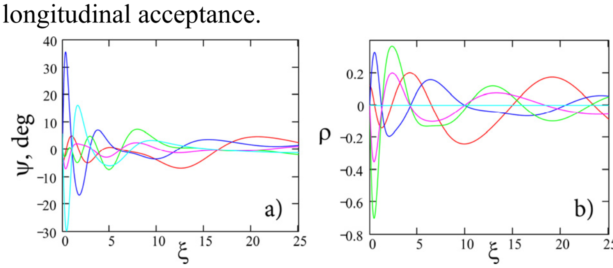

Figure 2: The longitudinal and transverse phase trajectories of the particles during the acceleration.

The beam dynamics numerical simulation was performed for the injection energy of 50 keV/u and the solenoid magnetic field 10 T and cavity potential 1MV. Some particle deviation from the equilibrium particle phase ψ and the change of the reduced radius ρ for several arbitrary particles with different initial simulation parameters are shown in Figure 2. It is clear from figure that the phase fluctuations are damped and the amplitude of transverse oscillations remains constant.COMputer simulation results

Analytical results obtained above were used to investigate the beam matching possibility at the linac output (see Fig. 3). For that the analytical results to be verified by computer simulations. The unbunched beam of 50 keV/u ions $^{132}Sn^{2+}$ with charge-to-mass ratio $Z/A = 1/66$ was studied. The initial radial deviation was 3 mm at most, $L = 1$ m, $L_r = 0.25$ m, $L_s = 0.2$ m, $B = 12$ T, $\varphi_1 = -30°$, $\varphi_2 = 20°$, $f = 57.5$ MHz, $\beta_G(0) = 0.01$, $eU/E_0 = 1$, period structure amount was equal to 25.

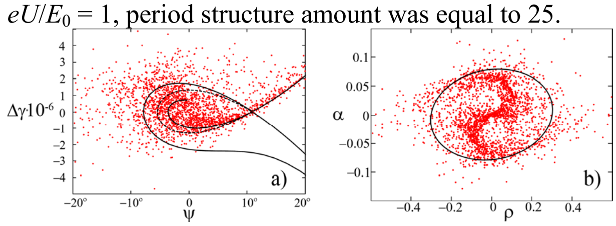

Figure 3: Longitudinal and transverse phase space.

The longitudinal spread of beam particles at the end of accelerator is shown dotted curve defines the longitudinal channel acceptance without dissipative effects and the solid line defines that taking into account decaying oscillations. One can see that the longitudinal acceptance in the latter case is greater than in the former one. The transversal spread of beam particle at the end of accelerator in represented Figure 3 b. The spread ellipse which confines 95% of particles within radius value 3 mm is also plotted in this figure.

NUMERICAL SIMULATION IN POLYHARMONIC FIELD

The numerical simulation of beam dynamics in polyharmonic field was performed to verify of the result obtained bellow. In this case we must take into account the slipping of particles relative to the accelerating wave. The acceleration rate reduces for this system and the beam longitudinal and transverse dynamics decay. If the geometric rate in each resonator will close to the speed of the quasi-equilibrium particle this effect is not observed. The comparison of equilibrium particles speed changes during acceleration is shown on Figure 4: the red line shows the change of velocity of the quasi-equilibrium particles obtained in the smooth approximation; the blue line show particle oscillations accelerated in a structure consisting of independent-phase cavities sequence. If we require that the amount of slipping factor does not exceed the acceptable limit (in this case is 20%), the velocity change will be slowly increase, then if the particle velocity close to β_G, the acceleration rate equal to the previous case, and when the difference between the beam velocity and phase velocity of the accelerating harmonic β_G will have large deviation becomes constant (Fig. 4, green curve).

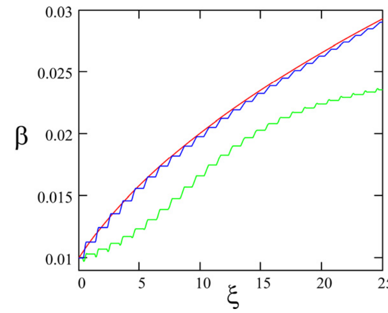

Figure 4: Equilibrium particle velocities during acceleration.

3D NUMERICAL SIMULATION TAKEN INTO ACCOUNT COULOMB FIELD

The beam dynamics simulation results in the case of a combined APF with an external magnetic field taking into account the Coulomb field are shown in Figure 5. As can be seen from figure the beam is in the separatrix and 95% of the particles not exceed a radius of 3 mm, which confirms the correctness of the accelerator parameters carried out previously.

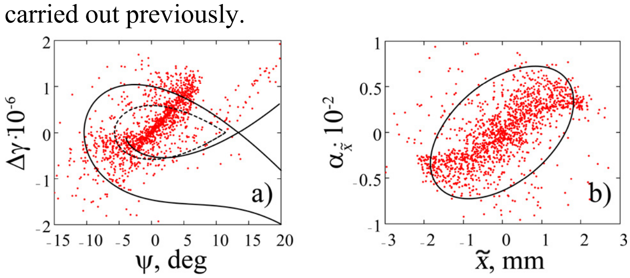

Figure 5: Longitudinal and transverse phase space in case of combined focusing

CONCLUSION

It was shown that the smooth approximation allows one find necessary restrictions on the beam dynamics. Analytical results are derived and verified by means of numerical simulations. All predicted results ere confirmed.

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