SVD-BASED FILTER DESIGN FOR THE TRAJECTORY FEEDBACK OF CLIC

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Abstract

The trajectory feedback of the Compact Linear Collider (CLIC) is an essential mitigation method for ground motion effects at CLIC. In this paper significant improvements of the design of this feedback are presented. The new controller is based on a singular value decomposition (SVD) of the orbit response matrix to decouple the in- and outputs of the accelerator. For each decoupled channel one independent controller is designed by utilising ground motion and noise models. This new design allows a relaxation of the required resolution of the beam position monitor from 10 to 50 nm. At the same time the suppression of ground motion effects is improved. As a consequence, the tight tolerances for the allowable luminosity loss due to ground motion effects in CLIC can be met. The presented methods can be easily adapted to other accelerators in order to loosen sensor tolerances and to efficiently suppress ground motion effects.

INTRODUCTION

CLIC is a proposal of CERN for a future high-energy particle collider. The luminosity performance of CLIC is sensitive to ground motion. Ground motion misaligns accelerator components, most importantly quadrupole magnets (QPs), which lead to emittance growth and beam-beam offset at the interaction point.

To address the ground motion problem, we present in this work a novel design method for trajectory feedbacks. The new design adds significant improvements compared to the last version [1]. It is based on a singular value decomposition (SVD) controller, which is a well-known strategy for trajectory control problems (e.g. [2]). The novelty of the design method is a semi-automatic procedure, which determines the controllers of the decoupled channels. In the first step, the user specifies a time-dependent filter $g(z)$, which forms the basis of all control loops. In the second step, one additional gain factor f_i per channel is determined by an L^2-minimisation procedure. This optimisation takes into account models of the ground motion, BPM noise and other ground motion mitigation methods. Since the f_i correspond to specific directions of the measurement vector, the calculation of them will be referred to as spatial filter design. The procedure leaves sufficient design freedom for the user by the choice of $g(z)$, and disburdens him/her from the tedious task of choosing the controller of each loop individually. This speeds up the design process significantly, while at the same time the feedback performance is improved.

The presented method is applied to the trajectory feedback design for the main linac (ML) and the beam delivery system (BDS) of CLIC. The trajectory feedback can only preserve the luminosity, in combination with the other three ground motion mitigation methods of CLIC: IP feedback, QP stabilisation and final focus stabilisation. Since the interplay of all these systems is hard to model by simple analytical formulas, full-scale simulations were set up. The results of these simulations are separately presented in [3] and only the outcomes, which are important to evaluate the new design method will be rephrased.

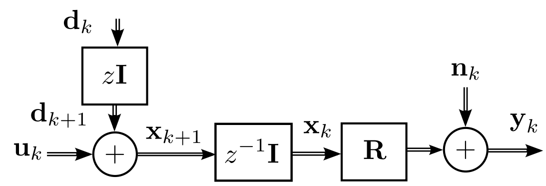

Figure 1: Block diagram of the model describing the beam oscillations in the ML and BDS. The unity matrix is symbolised by $\boldsymbol{I}$. The details are explained in the text.

ACCELERATOR SYSTEM

The trajectory feedback has to control the beam oscillations along the ML and BDS of CLIC excited by QP displacements (see Fig. 1). Since the accelerator is a discrete-time system, the so-called z-transform is used for its representation (see [4] for an introduction). The z-transform transforms a discrete-time signal or system into its frequency representation and is therefore analogous to the Laplace-transform, used for continuous systems. An important property of the z-transform, used in this text, is that a multiplication with z in the frequency domain corresponds to a unit time shift in the time-domain.

The 2104 QP positions at the next time step $\boldsymbol{x}_{k+1}$ (where k is the time index) are influenced by the ground motion $\boldsymbol{d}_k$ and the actuator settings $\boldsymbol{u}_k$. Note that the ground motion acts directly on the QP position, contrary to the time-shifted controller settings $\boldsymbol{u}_k$. To be able to use the standard control engineering system representation, in which the disturbance (ground motion), acts on the input, the time-shifted $\boldsymbol{d}_{k+1}$ is used instead of $\boldsymbol{d}_k$. The actuators that move the QPs to the controller settings $\boldsymbol{u}_k$ are so-called tripods. A tripod is a positioning device consisting of piezo-actuated

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legs, which support the QP. By altering the length of its legs, a tripod can move a QP mechanically (see [5]).

As the beam arrives, the QP positions $\boldsymbol{x}_k$ result in beam oscillations (multiplication with the response matrix $\boldsymbol{R}$). These beam oscillations are measured by the 2122 beam position monitors (BPMs) $\boldsymbol{y}_k$, where $\boldsymbol{n}_k$ is the measurement noise. The trajectory feedback algorithm uses these BPM measurements $\boldsymbol{y}_k$ to calculate corrector settings for the next time step $\boldsymbol{u}_{k+1}$. These actuator settings are supposed to steer the beam back onto its nominal trajectory $\boldsymbol{r}_0$. Characteristics of the accelerator system are its large size (2104 inputs and 2122 outputs) and its relatively simple structure without internal back coupling.

CONTROLLER DESIGN

Decoupling

The trajectory feedback of CLIC is a special form of a decoupling controller, called SVD controller [6]. The principle of a decoupling controller is the conversion of a multi-input, multi-output system into a new system, in which every input acts only on one output. For each of the decoupled system channels an independent single-input, single-output controller can be designed. This splitting of one large control problem into many smaller ones simplifies the design procedure significantly. In the case of a SVD controller, the decoupling is achieved with the help of the singular value decomposition (SVD) of the response matrix $\boldsymbol{R} = \boldsymbol{U\Sigma V}^T$, where $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthonormal matrices. An important property of an orthonormal matrices $\boldsymbol{A}$ is that $\boldsymbol{A}^T\boldsymbol{A} = \boldsymbol{I}$. Furthermore, $\boldsymbol{\Sigma}$ is a diagonal matrix with the singular values σ_i as elements. If the system in Fig. 1 is pre-multiplied with $\boldsymbol{V}$ and post-multiplied with $\boldsymbol{U}^T$, the new, decoupled system

$$\boldsymbol{U}^T z^{-1}\boldsymbol{I}\boldsymbol{U\Sigma V}^T\boldsymbol{V} = z^{-1}\boldsymbol{\Sigma} \tag{1}$$

is formed. The inputs and outputs of the new system ($\hat{\boldsymbol{u}}_k$ and $\hat{\boldsymbol{y}}_k$) do not correspond to individual tripods and BPMs anymore, but to whole input and output vector directions, given by the columns of $\boldsymbol{U}$ and $\boldsymbol{V}$. Consequently, also the ground motion and the BPM noise have to be transformed to $\hat{\boldsymbol{d}}_k = \boldsymbol{V}^T\boldsymbol{d}_k$ and $\hat{\boldsymbol{n}}_k = \boldsymbol{U}^T\boldsymbol{n}_k$. The SVD controller is especially well suited for trajectory control, since the accelerator system is large and therefore simplifications are necessary. These simplifications are achieved by the decoupling ability of the SVD controller. Since the internal structure of the system is simple, the decoupling is valid for all frequencies, which is usually not achievable.

For each of the decoupled channels an individual controller of the form $g(z)f_i/\sigma_i$ is designed, where i is the channel index. The division by the singular value σ_i corresponds to a normalisation of the loop gain in Eq. (1). The time-dependent filter $g(z)$ is chosen the same way for all controllers to reduce the number of parameters. Additionally one gain factor f_i is left open per channel to account for the different ground motion and BPM noise for each channel. The complete system is visualised in Fig. 2. Note that

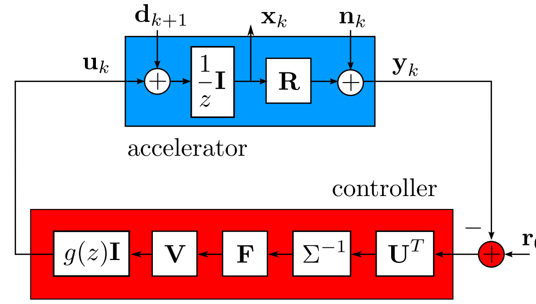

Figure 2: Block diagram of the trajectory feedback system. The coefficients f_i are collected in the diagonal matrix $\boldsymbol{F}$. The normalisation of the loop gains is achieved by the multiplication with $\boldsymbol{\Sigma}^{-1}$.

in this block diagram $g(z)\boldsymbol{I}$ and $\boldsymbol{V}$ are interchanged, due to commutativity, to separate the overall controller into two parts. One controller part is only time-dependent ($g(z)\boldsymbol{I}$) and one only depends on the direction of the input vector, which is called spatial filter ($\boldsymbol{VF\Sigma}^{-1}\boldsymbol{U}^T$).

Time-dependent Filter

The time-dependent filter $g(z)$ is composed of 4 parts.

$$g(z) = I(z)T(z)P(z)L(z). \tag{2}$$

The individual elements are given by

$$I(z) = \frac{z}{z-1}, \tag{3}$$

$$T(z) = \frac{z\left(1 - e^{-\frac{T_d}{T_1}}\right)}{z - e^{-\frac{T_d}{T_1}}} \tag{4}$$

$$\text{with} \quad T_d = 0.02\,\text{s} \quad \text{and} \quad T_1 = 0.1\,\text{s},$$

$$P(z) = \frac{(1-n_1)(1-n_2)}{(1-z_1)(1-z_2)}\frac{(z-z_1)(z-z_2)}{(z-n_1)(z-n_2)} \tag{5}$$

$$\text{with} \quad z_{1,2} = e^{(-1.43\pm 2\pi i 0.2)T_d}$$

$$\text{and} \quad n_{1,2} = e^{(-0.3\pm 2\pi i 0.3)T_d},$$

$$L(z) = \frac{(1-n_3)(z-z_3)}{(1-z_3)(z-n_3)} \tag{6}$$

$$\text{with} \quad z_3 = e^{-17T_d} \quad \text{and} \quad n_3 = e^{-38T_d}.$$

The design of these filters is performed with the classical loop-shaping method (see [6]). In the following only the purpose but not the design of the individual components is explained. The integrator $I(z)$ is the key element of $g(z)$. It adds up the increments calculated by the spatial filter. Even though $I(z)$ suppresses ground motion well, it also couples the BPM noise $\hat{\boldsymbol{n}}(z)$ too strong into the system. Therefore, the low pass $T(z)$ is added to the design to improve the noise behaviour. The element $P(z)$ has to be added, since the final doublet QPs and the other QPs are stabilised by different transfer functions (passive damper and QP stabilisation), which differ strongly around 0.3 Hz. To account for the strong mismatch of these transfer functions around 0.3 Hz, the controller is strengthened in this frequency range. This is accomplished by $P(z)$, which amplifies frequencies around 0.3 Hz and leaves other frequency components unchanged. The so-called lead-element $L(z)$ is added for stability reasons. An important measure for the stability of a control circuit is the phase margin to -180° of the open loop transfer function, at the cross over frequency (Nyquist criterion). The combination of $I(z)$, $T(z)$ and $P(z)$ leads to an insufficient phase margin. To improve the stability properties $L(z)$ is added to lifts the phase, which results in a phase margin of 36.3°.

Spatial Filter

For each controller loop there is still one gain parameter f_i, which can be chosen to minimise the output signal $\hat{y}_{k,i}$, with respect to the system excitation. The index i is skipped in the following, to make the expressions more concise. The frequency representation of $\hat{y}_k$ is given by

$$\hat{Y}(i\omega) = \hat{S}(z = e^{i\omega T_d})\hat{D}(i\omega) - \hat{T}(z = e^{i\omega T_d})\hat{N}(i\omega), \quad (7)$$

where we use that the z-transform of a system evaluated at $z = e^{i\omega T_d}$ corresponds to the transfer function of the system. The ground motion suppression and noise z-tranform $\hat{S}(z)$ and $-\hat{T}(z)$ of the system are given by

$$\hat{S}(z) = z\frac{\hat{G}(z)}{1+\hat{G}(z)\hat{C}(z)}, \quad \hat{T}(z) = \frac{\hat{G}(z)\hat{C}(z)}{1+\hat{G}(z)\hat{C}(z)} \quad (8)$$
$$\text{with} \quad \hat{G}(z) = \frac{\sigma}{z} \quad \text{and} \quad \hat{C}(z) = g(z)\frac{f}{\sigma},$$

where $\hat{G}(z)$ and $\hat{C}(z)$ are the z-transformed of the decoupled system channel and its associated controller. The channel ground motion spectrum $\hat{D}(i\omega)$ can be calculated by using existing ground motion models, the geometry and response matrix $\boldsymbol{R}$ of the accelerator. The derivation is omitted here, due to space limitations. The decoupled noise spectrum $\hat{N}(i\omega)$ can be modelled as white noise (flat spectrum). The according variances are given by the diagonal elements of the expression $E\left\{\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}^{\boldsymbol{T}}\right\} = \boldsymbol{U}^T E\left\{\boldsymbol{n}\boldsymbol{n}^T\right\}\boldsymbol{U}$, where $E\{.\}$ is the expectation value of a random variable. To find the optimal value of f, the expression $E\{\hat{y}_k(f)\}$ is minimised for each channel independently. This is equivalent to minimise of the L^2-norm of the spectrum of $\hat{y}_k$

$$\min_f ||\hat{Y}(i\omega, f)||_{L^2} = \min_f \int_{\omega=-\infty}^{+\infty} |\hat{Y}_i(i\omega, f)|^2 d\omega. \quad (9)$$

Eq. (9) is solved numerically, by evaluating the integral for different values of f over a sufficient frequency range.

RESULTS

Full-scale simulations showed, that the new trajectory feedback (together with the other three ground motion mitigation methods) is capable of preserving the luminosity of CLIC. The luminosity is decreased by ground motion to a value that is still 10 % better than the design specification

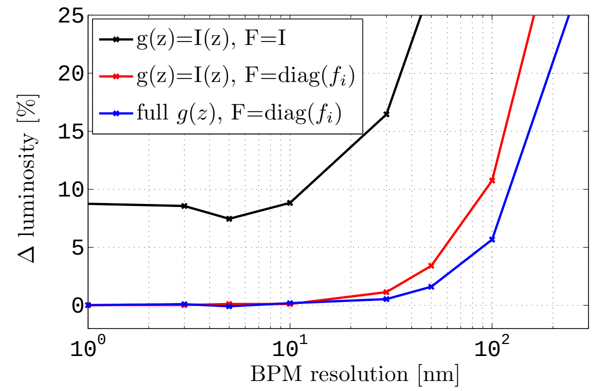

Figure 3: The graphs show, how different controller variants (full $g(z)$ means $g = I(z)T(z)P(z)L(z)$) influence the luminosity loss due to the BPM resolution.

(see [3] for more details). This important result could only be achieved, due to the high robustness of the trajectory feedback to BPM noise (see Fig. 3). As a result the BPM resolution tolerances have been loosened from 10 to 50 nm.

CONCLUSION

In this paper we present a design method of trajectory feedbacks. The method is based on a SVD decoupling of the in- and outputs and adds one design parameter f_i per decoupled channel. Each of these 2104 f_i is adapted using a L^2-minimisation problem, to balance the influence of ground motion, QP stabilisation and BPM noise optimally. The parameters f_i are gain factors for a user-defined, frequency-dependent filter $g(z)$ that is applied to all channels in order to simplify the design procedure. With the semi-automatic technique of first choosing $g(z)$ and then optimising the 2104 f_i, expert knowledge can be incorporated in the design. The method was tested for the feedback design of the ML and BDS of CLIC. The use of the new feedback results in 1.5 % luminosity increase, compared to the old, tediously hand-optimised controller. The new feedback is also very robust against noise, which led to a relaxing of the tight BPM tolerances for CLIC. Concluding we can state that the presented method is not only improving the performance of the designed trajectory feedbacks, but also significantly reduces the design time.

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