

MUON ACCELERATION WITH RLA AND NON-SCALING FFAG ARCS*

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Abstract

Recirculating Linear Accelerators (RLA) are the most likely means to achieve the rapid acceleration of short-lived muons to multi-GeV energies required for Neutrino Factories and TeV energies required for Muon Colliders. In this paper, we present a novel return-arc optics design based on a Non Scaling Fixed Field Alternating Gradient (NS-FFAG) lattice that allows 5 and 9 GeV/c muons of both charges to be transported in the same string of magnets. The return arcs are made up of super cells with each super cell consisting of three triplets. By employing combined function magnets with dipole, quadrupole, sextupole and octupole magnetic field components, each super cell is designed to be achromatic and to have zero initial and final periodic orbit offsets for both 5 and 9 GeV/c muon momenta. This solution would reduce the number of arcs by a factor of 2, simplifying the overall design.

INTRODUCTION

Figure 1 shows a proposed [1] dog-bone-shaped muon RLA consisting of a single linac with pulsed quads and separate droplet return arcs. In the example illustrated in Fig. 1, the number of passes is increased by pulsing linac quads from 8 passes to 12, leading to significant cost savings. However, in that scheme, one needs to separate different energy beams coming out of the linac and to direct them into appropriate droplet-shaped arcs for recirculation. Each pass through the linac would call for a separate fixed energy droplet arc, increasing the complexity of the RLA. Here, we propose a novel return-arc optics design based on a Non-Scaling Fixed Field Alternating Gradient [2] (NS-FFAG) lattice, which allows two (potentially even more) consecutive passes with very different energies to be transported through the same string of magnets.

DROPLET ARC REQUIREMENTS

Through use of large momentum acceptance NS-FFAG arcs in a dogbone RLA, one can maximize the number of passes that μ^\pm can be accelerated through a single linac. The arc layout is similar to the separated arcs structure in Fig. 1. Each droplet arc consists of a 60° outward bend, a 300° inward bend and another 60° outward bend so that the net bend is 180°. This arc geometry has the advantage that if the outward and inward bends are made up of similar cells, the geometry automatically closes without the need for any additional straight sections, thus making it simpler and more compact.

To transport different energy muons of both charges through the same arc structure, the arc must possess the following properties:

- 1) For each transported momentum, the periodic orbit's offset must be zero at the arc's entrance and exit to ensure that the beam goes through the center of the linac.
- 2) The arc must be achromatic for each momentum to guarantee matching to the linac.
- 3) The arc must be mirror symmetric, so that μ^+ and μ^- can pass through the same lattice in opposite directions. The symmetry ensures that the periodic beta functions are identical at the arc's ends and that the periodic alpha functions and dispersion slope are zero at the ends.
- 4) The arc must be near isochronous for both energies to ensure proper phasing with the linac.
- 5) The orbit offsets as well as beta functions and dispersion for both energies should be small enough to keep the aperture size acceptable.

This proposed NS-FFAG lattice meets all these requirements.

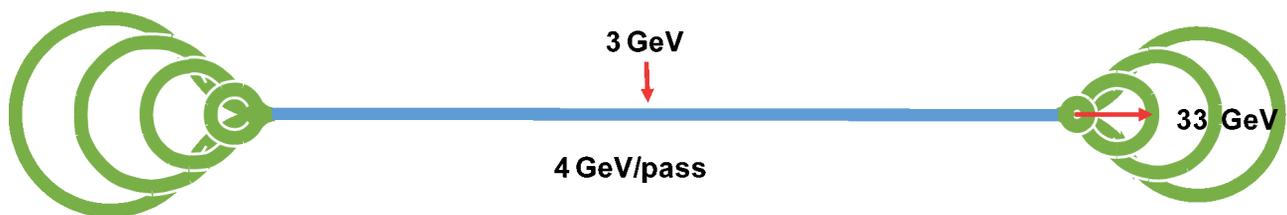


Figure 1: Layout of an 8-pass ‘Dogbone’ RLA with the top-to-injected energy ratio of 11.

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ARC DESIGN BASED ON NS-FFAG CELL

As a basis of our droplet arc design, we used the NS-FFAG [2] triplet magnet arrangement, which was extensively studied in [3]. The outward-bending triplet cell consists of an inward bending combined function magnet with positive gradient (horizontally focusing) at the center and two outward bending magnets located on either side with equal negative gradients. The inward-bending triplet cell has the same structure but reversed dipole fields. The fact that the cells are symmetric with respect to their centers ensures that their periodic solutions have $\alpha_x = \alpha_y = 0$ and $D'_x = 0$ at the beginning and the end of the cells. It was demonstrated in [3] that, by using combined function magnets, the arc structure can be made very compact and that such a lattice can accommodate beam energies over a very large range and is characterized by small beta functions and small dispersion due to the strong focusing of a NS-FFAG.

The study reported in [3] considered combined function magnets with dipole and quadrupole magnetic field components only. It was found that, despite a large momentum acceptance, the off-momentum periodic orbit's offset and the off-momentum periodic dispersion were not zero at the entrance and exit of the triplet cells making matching the cells to the linac and matching the outward bending cells to the inward bending cells difficult. Thus, in our study we used combined function magnets, which in addition to the dipole and quadrupole components, also included sextupole and octupole ones.

To study the optics of the NS-FFAG structure for large momentum range, we used the Polymorphic Tracking Code (PTC) module of the MAD-X program [4]. While perturbative method codes are not suitable for such a study, PTC allows symplectic integration through all elements with user control over the precision (with full or expanded Hamiltonian).

For simplicity we made 5 GeV/c the nominal momentum going through the magnet centers. The constraint that the 5 GeV/c periodic orbit has to have zero offset coming in and out of the cell is then automatically satisfied. Besides, once the 5 GeV/c linear optics is adjusted with quadrupole gradients, introduction of the sextupole and octupole magnetic field components required for accommodating the 9 GeV/c momentum does not change it. This decouples the 5 GeV/c linear optics from the 9 GeV/c optics and ensures that, once the $D_x = 0$ and $D'_x = 0$ conditions are satisfied with the quadrupole gradients at 5 GeV/c, they are not affected by tuning of the 9 GeV/c linear optics with the sextupole and octupole components.

For the triplets, we chose 1 m long magnets separated by 20 cm gaps. To simplify the geometry, each magnet's bending angle was set to 5° . We then adjusted the quadrupole gradients to make the triplet cell achromatic. The quadrupole gradient of the middle magnet was adjusted as strong as possible without losing transverse motion stability in order to minimize the 9 GeV/c orbit offset. This determined the cell's phase advance. Figures 2 and 3 show the 9 GeV/c period orbit, beta functions, and dispersion for the outward and inward bending cells, respectively. Note that, since there is no coupling in our case, the Ripken's β_{11} and β_{22} are simply equal

respectively to the usual horizontal and vertical beta-functions. Comparing Figs. 2 and 3, one can see that the beta functions for the two cell types are the same while the dispersion changes sign. Since the cells are achromatic, they can be matched together in a natural way.

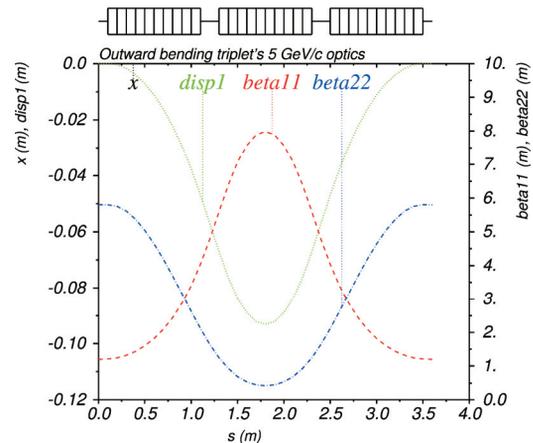


Figure 2: 5 GeV/c periodic orbit, dispersion and beta functions of the outward bending triplet cell.

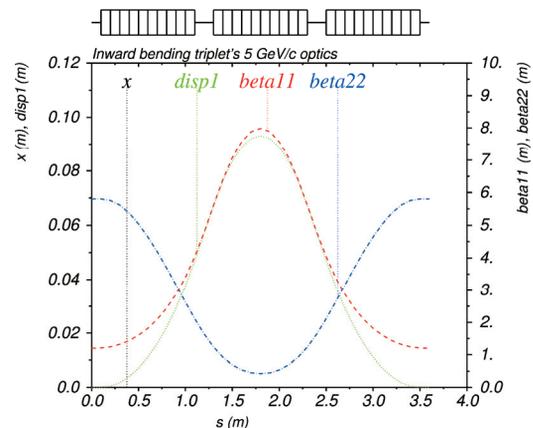


Figure 3: 5 GeV/c periodic orbit, dispersion and beta functions of the inward bending triplet cell.

We next studied the triplet cell's 9 GeV/c optics by introducing sextupole components in the cell's magnets in a symmetric way. The 9 GeV/c periodic orbit's deviation is large in the middle magnet and is much smaller in the side magnets. Therefore, the effect of the sextupole component is large in the middle magnet and is almost negligible in the side magnets. For this reason, it was not possible to satisfy, at the same time, the zero orbit offset and achromatic conditions by adjusting the sextupole components of the middle and side magnets.

Therefore, we combined three triplet cells into a super cell. The sextupole strength of the middle magnets of the two outer triplet cells was used as one parameter. The sextupole strength of the middle magnet of the central triplet cell was the second parameter. This arrangement preserves the super cell's mirror symmetry. By varying the above two parameters, we were able to simultaneously satisfy the conditions of the super cell being achromatic and having zero incoming and outgoing periodic orbit offset. To keep the transverse motion stable in both

dimensions, we added small octupole components of the same strength to the center magnets of all three triplets. This modified the field gradient along the 9 GeV/c reference orbit in these magnets restoring the stability. The periodic orbit and dispersion of the outward bending super cell are shown in Fig. 4. Figure 5 illustrates the 9 GeV/c beta functions of that type of cell. Figures 6 and 7 show similar graphs for the inward bending super cell. Figures 4-7 demonstrate that changing the bending direction does not affect the beta functions, but reverses the signs of the periodic orbit and dispersion. Since the super cell is achromatic and has zero incoming and outgoing periodic orbit offset, it is clear that the super cells are automatically matched. Since the net bend of each super cell is 15° , they can be easily put together to form the 60° and 300° bends of the droplet arc.

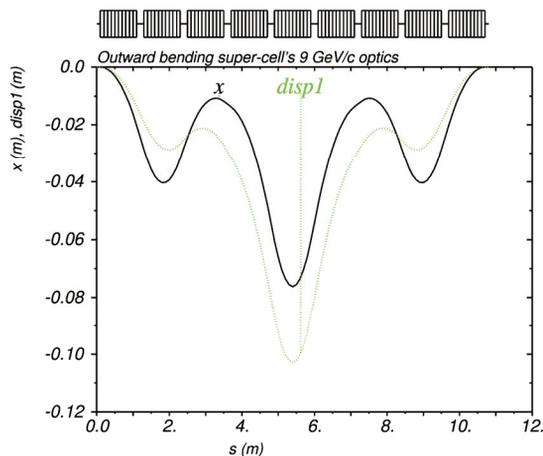


Figure 4: 9 GeV/c periodic orbit and dispersion of the outward bending super cell.

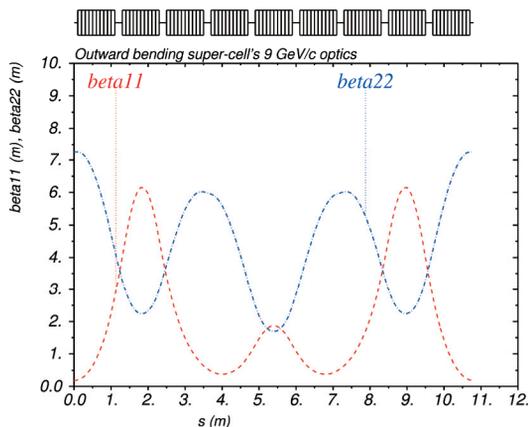


Figure 5: 9 GeV/c beta functions of the outward bending super cell.

This design can be further optimized to minimize the orbit offset and the required magnetic fields. One can consider including more triplets in the super cell. Another path for improvement is to choose the reference momentum to be between 5 and 9 GeV/c. This would also allow one to adjust the path lengths to make the arc closer to isochronous for the different momenta. The non-linear effects and the dynamic aperture are being studied.

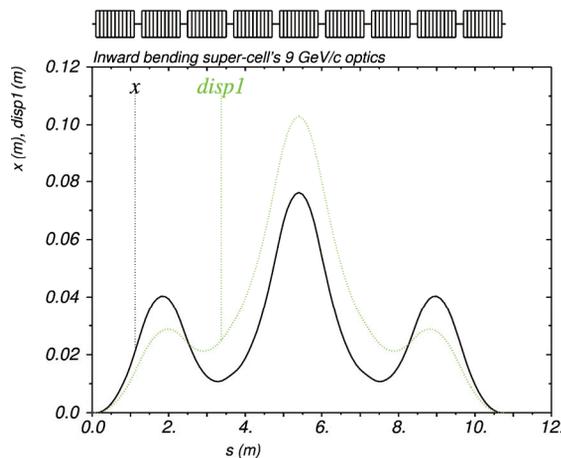


Figure 6: 9 GeV/c periodic orbit and dispersion of the inward bending super cell.

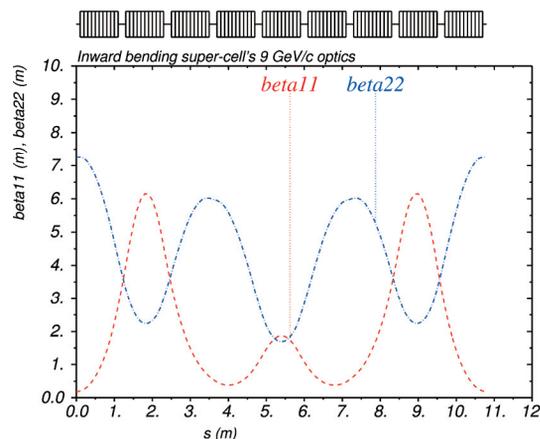


Figure 7: 9 GeV/c beta functions of the inward bending super cell.

CONCLUSION

A droplet arc design based on a NS-FFAG lattice has been developed to transport both 5 and 9 GeV/c muon beams of both charges. This lowers the number of arcs and eases the design of a muon RLA. The arc properties of being achromatic and having zero periodic orbit offset for the two momenta facilitates matching of the arc to the linac. As the next step, we plan to study matching of our NS-FFAG arc to the multi-pass linac optics by lifting the symmetry requirements in some of the cells adjacent to the droplet arc ends and using the additional parameters to satisfy the matching conditions.

REFERENCES

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