SIX-DIMENSIONAL COOLING LATTICE STUDIES FOR THE MUON COLLIDER[*]

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Abstract

A significant reduction in the six-dimensional emittance of the initial beam is required in any proposed Muon Collider scheme. Two lattices based on the original RFOFO ring design representing different stages of cooling are considered. One is the so-called open cavity lattice addressing the problem of the 201.25 MHz RF cavities running in a magnetic field, the other one is the 805 MHz RF lattice that is used for smaller emittances. The details of the acceptance analysis and tracking studies of both channels are presented and compared to the independent ICOOL implementation.

RFOFO-BASED COOLING LATTICES

In a Muon Collider design the muon beam 6D phase space volume must be reduced several orders of magnitude in order to be able to further accelerate it. Ionization cooling is currently the only feasible option for cooling the beam within the muon lifetime ($\tau_0 = 2.19$ μs). The RFOFO ring [1] is one of the feasible options along with other designs. The RFOFO ring provides a significant reduction in the six-dimensional emittance in a small number of turns with a relatively low particle loss factor. 6D cooling is achieved by employing the concept of emittance exchange. When a dispersive beam passes through a wedge absorber in such a way that higher momentum particles pass through more material, both the longitudinal and the transverse emittances are reduced. However, the design of the injection and extraction channels and kickers is very challenging for the RFOFO, and the ring could not be used as is, because the bunch train is too long to fit in the ring. Both problems are mitigated in the RFOFO helix, also known as the Guggenheim channel [2]. In addition, using the helix solves another important issue, namely, the overheating in the absorbers. The Guggenheim performance is comparable to the original RFOFO ring.

The layout of the RFOFO ring is shown in Fig. 1.

Various studies suggest that the presence of the magnetic field disrupts the performance of RF cavities by causing breakdown [3]. Thus, it was proposed to consider an alternative layout of the cooling channel, the so-called open cavity lattice [4]. The concept itself consists of two parts: a) moving the solenoidal coils from over the RF cavities into the irises; and b) shaping the RF cavities such that the walls of the cavities are predominantly parallel to the magnetic field lines, which hopefully solves the problem of the breakdown. The layout simulated in G4Beamline is illustrated in Figure 2. It does not include specifically shaped RF cavities; instead a simplified pillbox geometry is used.

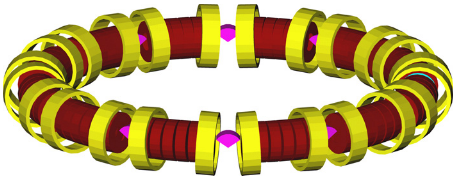

Figure 1: RFOFO ring layout. Yellow—tilted magnetic coils with alternating currents to provide necessary bending and focusing and generate dispersion, purple—wedge absorbers for cooling and emittance exchange, brown—RF cavities for restoring the longitudinal component of the momentum.

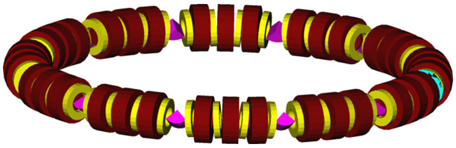

Figure 2: Open cavity layout. Color-coding is the same as for Fig. 1.

The open cavity cooling channel layout has 12 cells with three RF cavities in each and four solenoidal coils in the irises. The circumference of the ring is 30.72 meters. The idea of tipping the solenoids, similar to the RFOFO ring concept, is employed in this layout to generate an average vertical magnetic field of 0.136 T providing necessary bending. Solenoid axes are tilted 4.9° above or below the orbital midplane depending on the direction of the current. The centers of the solenoids are displaced radially outward from the reference circle by 21 mm to minimize the integrated on-axis radial field and thus vertical beam deviations.

The open cavity cooling lattice overall performance is very similar to that of the original RFOFO lattice as evidenced by Fig. 3 and Table 1.

ACCEPTANCE ANALYSIS

Both cooling channels exhibit the same transmission loss problem: approximately 21% of particles are lost in the beginning in a typical simulation with stochastic processes on but no decay. These losses are illustrated in Fig. 4. In an attempt to categorize transmission losses and find a beam

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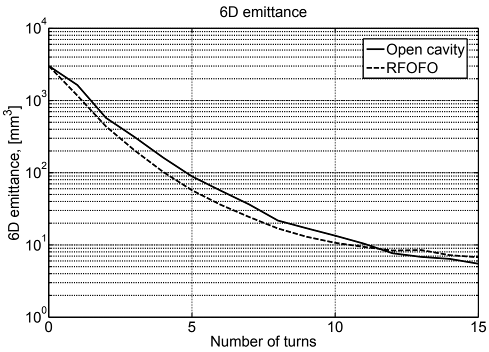

Figure 3: Six-dimensional emittance reduction in the open cavity lattice over 15 turns (461 meter).

Table 1: Performance of the open cavity ring compared to the original RFOFO ring.

Structure	$\varepsilon_\perp$ [mm]	$\varepsilon_\parallel$ [mm]	ε_{6D} [mm^3]	Transmission [%]
Initial	12	19	3000	100
Open cavity (461 meters)	1.5	2.3	5.5	57
RFOFO (462 meters)	1.7	2.5	7.2	56

that is matched to the cooling lattice, acceptance analysis of the RFOFO cooling lattice has been performed.

For this analysis the initial distribution is taken to be one million muons in six dimensions (three coordinates and three momenta) uniformly distributed in a cube, the size of which is deliberately larger than the acceptance of the channel. These particles are run through the RFOFO channel for ten turns to ensure all the particles that are lost have been properly accounted for. Judging from the previous simulations the proper matching of the beam to the RFOFO structure takes about five turns, so ten turns should be more than enough, which is confirmed by minimal losses after five turns observed in the simulations for this study.

After the first step about 65000 particles survive out of one million of the initial distribution. This is the starting point of the iterative process. These surviving particles are run again for another ten turns with a different random number generator seed to assure the randomness of the stochastic processes occurring. Indeed, not all the particles that survived the first step survive the second; thus we get a further refinement. The idea is to repeat the process enough times in order to obtain a distribution that suffers small losses in the simulation with stochastic processes on.

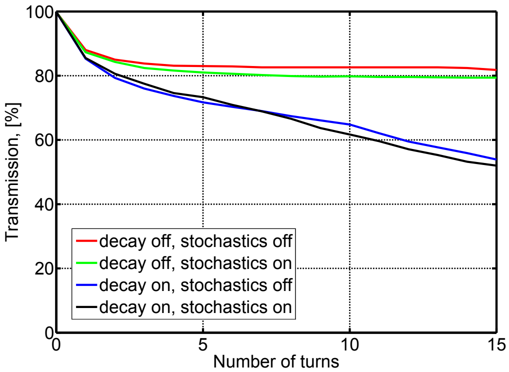

Figure 4: Transmission losses in the RFOFO cooling lattice.

Table 2: Parameters of the typical distribution used for RFOFO simulations *vs.* the acceptance study surviving distribution.

	Typical distribution	Acceptance study final distribution
σ_x, [mm]	41.79	55.43
σ_y, [mm]	42.86	45.66
σ_{p_x}, [MeV/c]	29.98	32.45
σ_{p_y}, [MeV/c]	30.07	28.84
σ_t, [ns]	0.298	0.376
σ_{p_z}, [MeV/c]	26.21	19.78

Particle loss stabilizes after 13 iterations at a steady rate of 4% with each new iteration. We consider it to be a reasonable stopping condition and assume the resulting distribution is a good approximation to the acceptance of the channel. Table 2 shows that $\sigma_{p_z} = 19.78$ MeV/c of the accepted distribution is significantly smaller than that of the typical distribution used for RFOFO simulations. Moreover, σ_{p_z} falls below the reference value of 26.21 MeV/c immediately after the first iteration. Hence, the longitudinal dynamics is the main source of particle loss. However, all other parameters such as the deviations in the two transverse planes and in time are still large enough after 13 iterations compared to the typical beam.

The distributions in the transverse coordinates are very close to being gaussian, while the distributions in the longitudinal coordinates are clearly asymmetric (see Fig. 5).

If one attempts to approximate the final distribution by a six-dimensional gaussian distribution preserving the correlations between variables, the approximate distribution still shows the loss of about 20% of particles after ten turns in the RFOFO channel with stochastics processes on and decay off. This implies that matching the second order moments alone is not sufficient, even if the approximate distribution is derived from the final distribution that shows only 4% loss at the next iteration, or that the longitudinal phase space cannot be accurately described as a gaussian.

805 MHZ COOLING LATTICE

Cooling efficiency is often characterized in terms of the quality factor

$$Q = \frac{d\epsilon_{6D}^N/ds}{dN/ds}\frac{N(s)}{\epsilon_{6D}^N(s)},$$

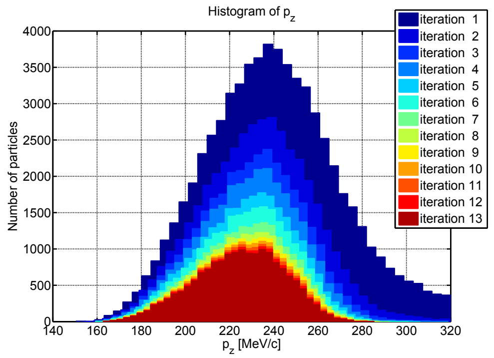

Figure 5: Longitudinal momentum distribution after 13 iterations of the acceptance analysis algorithm.

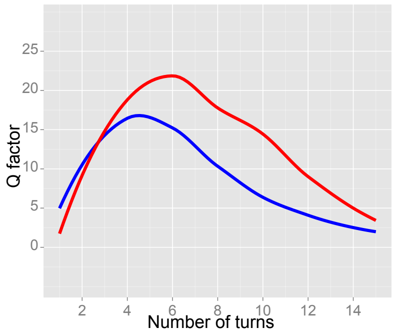

Figure 6: Q factor evolution in the original RFOFO lattice and the open cavity lattice.

where $\epsilon_{6D}^{N}(s)$ is the normalized six-dimensional emittance of the beam at arclength s and $N(s)$ is the number of surviving particles at arclength s. The Q factor compares the rate of change of emittance to the particle loss. Figure 6 demonstrates that as the number of turns grows the cooling efficiency decreases. In order to keep cooling rates high it is important to tune the parameters of the cooling channel accordingly. In general, that means smaller radius, stronger magnetic fields and higher frequency RF cavities toward the end of the channel. Ideally, the cooling channel should be tapered with various parameters changing slowly along the channel. However, changing the RF frequency continuously is very expensive, so the number of different frequencies should be as small as possible.

Before any tapering can be done in G4Beamline it is necessary to ensure that "boundary" lattices work properly. The two boundary cases are the 201.25 MHz RFOFO lattice in the beginning and its scaled version which smaller in size with RF cavities operating at 805 MHz and magnetic coils placed closer to the axis (see Fig. 7) used for smaller emittances. The 201.25 MHz lattice has been studied in detail [1, 2], the 805 MHz lattice has never been simulated in G4Beamline before.

Tracking simulation results for the 805 MHz lattice are summarized in Fig. 8. These results are consistent with an independent ICOOL simulation which shows that longitudinal emittance shrinks from 5 mm to 1.2 mm, transverse emittance shrinks from 1 mm to 0.38 mm, and 55% of particles survive with decay and stochastic processes taken into account [5]. The 805 MHz lattice is a challenging one for a number of reasons. First of all, the magnetic coils are very close to the axis and also to each other, especially when the coils are tilted 6° in order to generate the required bending field. As an immediate consequence, there is very little room for the absorber between the two coils, so that the ends of the absorber have to be cut. The strong magnetic field potentially creates problems similar to the original RFOFO, namely it can disrupt the RF cavity operation. These complications apart, the simulations show that the 805 MHz lattice is capable of cooling small emittances and the results are consistent between the two independent implementations.

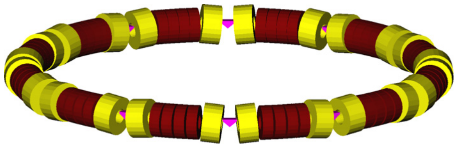

Figure 7: 805 MHz lattice layout. The circumference of the ring is 10.8 meters, and color-coding is the same as for Fig. 1.

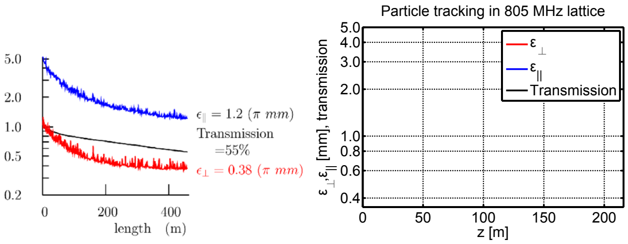

Figure 8: Particle tracking, 805 MHz cooling lattice. Blue—longitudinal emittance, red—transverse emittance, black—transmission. Top: ICOOL simulation, bottom: G4Beamline simulation.

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