# **NEW SCHEME OF QUASI-PERIODIC UNDULATORS**

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#### Abstract

Conventional quasi-periodic undulators are realized by introducing a smaller phase slip at appropriate positions in a periodic undulator. On the other hand, new type quasi-periodic undulators can be realized by introducing a larger phase slip. This new scheme gives possibilities to create new type magnetic configurations that provide additional freedom for manipulating a spectral feature.

## INTRODUCTION

The structure of original quasi-periodic undulator was realized by aligning magnetic poles at the positions of one-dimensional quasi-periodic lattice [1-4]. After the first prototype, a practical and effective modification of magnetic structure was proposed by groups of ESRF and ELETTRA [5]. In addition to this useful improvement, since a new parameter was introduced in the creation theory of one-dimensional quasi-periodicity, fabrication process became much easier [6,7]. As the result, many QPUs for generating both linearly and elliptically polarized radiations have been and will be installed in the synchrotron radiation (SR) facilities worldwide [8,9].

In this paper, the creation theory of 1D quasiperiodicity for the basis of QPU construction is reviewed, and a new method showing a guideline how to construct a magnetic structure is presented.

## **1-D QUASIPERIODICITY**

One of the easy-to-understand ways to create the onedimensional quasi-periodicity is to project lattice points in a window in the two-dimensional periodic lattice onto an irrationally inclined line. This procedure can be transformed into a simple equation as follows.

$$\widehat{z}_m = m + (r \tan \alpha - 1) \left[ \frac{\tan \alpha}{r + \tan \alpha} m + 1 \right].$$
(1)

In this equation,  $\tilde{z}_m$  represents a normalized coordinate of *m*-th lattice point on the inclined axis, and the bracket  $\lfloor x \rfloor$  stands for the greatest integer less than x. The letter *r* represents the ratio b/a [6,7].

Values of these coordinates are proportional to the phase advance of emitted light from the origin in an undulator, and therefore, it can be written as:

$$\phi_m = \pi \left\{ m + (r \tan \alpha - 1) \left[ \frac{\tan \alpha}{r + \tan \alpha} m + 1 \right] \right\}. (2)$$

The phase advance in each half period is written as:

$$\Delta \phi = \phi_{m+1} - \phi_m \,, \tag{3}$$

and those in a typical interval of periodic section and of quasi-periodic section are written as:  $\Delta \phi_n = \pi$  and

$$\Delta \phi_a = \pi r \tan \alpha$$
, respectively.

For introducing above mentioned two different phase slips to certain positions in actually constructed QPUs, the smaller phase advance is introduced at intervals of quasiperiodic positions by reducing peak magnetic field.

In general, the phase function for each half-period in an undulator is given by the following equation [10]:

$$\phi = \frac{\pi}{\lambda_{photon}} \left( \frac{z}{2\gamma^2} + \frac{\int x'^2 dz}{2} \right), \tag{4}$$

where  $\lambda_{\text{photon}}$  is the wavelength of emitted photon and x' is the angle of electron trajectory in an undulator.

After some simple calculations by assuming the sinusoidal magnetic field in each full-period, the phase slip ratio is found to be nearly equal to the most right part of equation (5):

$$\Delta \phi_q / \Delta \phi_p = \frac{\pi r \tan \alpha}{\pi} \cong \left( \frac{2B_{0q}^2 - B_{0p}^2}{B_{0p}^2} \right).(5)$$

As one can see in eq. (4), by selecting appropriate values for r and  $\tan \alpha$ , the phase advance at a quasiperiodic position can take larger value than that of periodic position, and hence the peak field at QP position can be larger than that at the periodic position. Fig. 1 shows such an example.

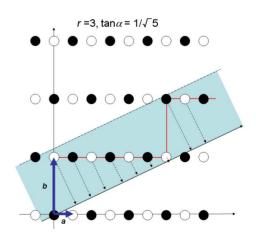


Figure 1: Example of new configuration of quasiperiodicity applied for a new type QPU.

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## **ONE-AXIS FIELD QPU**

Figure 2 shows the magnetic field distributions for the example shown above. In this example, the peak field at periodic positions were assumed to be 0.3 T, the 100 mm period length, and the 3.2 m undulator length. The deflection parameter K for periodic part is 2.8.

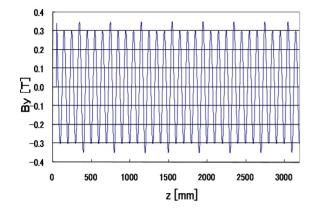


Figure 2: Magnetic field distribution of a new QPU using the quasi-periodic lattice described in Fig. 1.

Figure 3 shows expected on-axis flux density from undulator having the field in Fig. 2. For the spectral calculation, the HiSOR-II ring parameters, 0.7 GeV electron energy, 300 mA beam current, 14 nm-rad emittance with 1 % coupling were assumed [11].

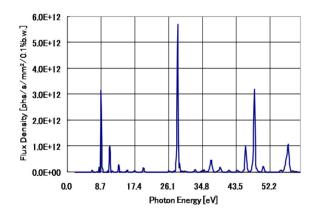


Figure 3: On-axis flux density spectrum of a new-type QPU with the field distribution in Fig. 2. Observation point locates at 30-m from the source.

As it can be clearly seen in this figure, in the new configuration of magnetic structure (increased field strength at the QP-positions), major higher harmonic peak positions shift from rational positions toward higher energy positions [12].

## **TWO-AXIS FIELD QPU**

As we see in the previous section, in order to generate irrational higher harmonics, it is necessary to adjust the magnetic field so that the phase slip at the QP-position becomes equal to  $\pi$ rtan $\alpha$ . Only the way to introduce a quasi-periodicity in conventional QPU has been to reduce the magnetic field in order to introduce smaller phase slip. However, by selecting an appropriate QP lattice, one can introduce larger phase slip at a QP position by increasing larger magnetic field and/or by introducing additional magnetic field orthogonal to original periodic field. The equation describes the relation between the phase slip and the strength of magnetic field is written as follows [13,14]:

$$\Delta \phi_q / \Delta \phi_p = \frac{\pi r \tan \alpha}{\pi} \cong \frac{B_{0y}^2 + B_{0x}^2}{B_{0y}^2}$$
(6)

Figure 4 shows the magnetic field distribution for such a two-axis field QPU using the same parameters as in the previous section.

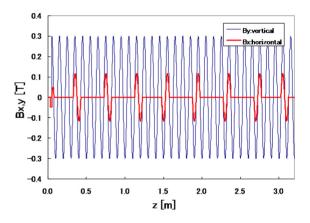


Figure 4: Magnetic field distribution of a new orthogonalfield quasi-periodic undulator. *L*=3.2 m,  $\lambda_u$ =100 mm,  $B_{0p}$ =0.3 T.

In this figure, the positions of horizontal field area are the same with the QP positions in Fig. 2.

Figure 5 shows expected on-axis flux density from this undulator.

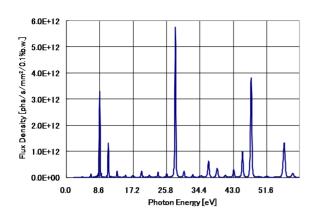


Figure 5: On-axis flux density spectrum of a new-type QPU with the field distribution in Fig. 4. Observation point locates at 30-m from the source.

This spectral shape is similar to that in Fig. 3 though the peak intensity of fundamental radiation in Feg. 5 is slightly higher.

# **DISCUSSIONS AND CONCLUSION**

Especially for the one-axis-field QPU, it is difficult to introduce the phase slip only in a half-period distance without introducing a phase error in perimetric area [14]. In order to solve this problem, an analytical treatment was slightly modified so that the overall phase slip in a full one-period becomes  $2\pi(d'+d)/2d$  at the area including a QP-position and adjacent two quarter periods [15].

In this paper, a new possibility to create different types of quasi-periodic undulator was proposed. An advantage of two-axis QPU is that one can construct a QPU by just adding a small number of necessary magnets corresponding to a one-period length at QP positions to a periodic undulator.

In conclusion, a possibility for designing a new type QPU is presented. There might be further possibilities for

creating QPUs with higher spectral performance by further modification of QP lattice. Possible realistic magnetic structure of new QPU will be presented elsewhere.

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