# A MUON COLLIDER SCHEME BASED ON FRICTIONAL COOLING

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## Abstract

Muon colliders would open new frontiers of investigation in high energy particle physics, allowing precision measurements to be made at the TeV energy frontier. One of the greatest challenges to constructing a muon collider is the cooling of a beam of muons on a timescale comparable to the lifetime of the muon. Frictional cooling holds promise for use in a muon collider scheme. By balancing energy loss to a gas with energy gain from an electric field, a beam of muons is brought to an equilibrium energy in 100s of nanoseconds. A frictional cooling scheme for producing high-luminosity beams for a muon collider is presented.

# INTRODUCTION

The problem of preparation of a muon beam on a timescale comparable to the lifetime of the muon is one of the main challenges in building a muon collider. Frictional cooling [1], a scheme for beam emmittance reduction, can be incorporated into a muon collider scheme to produce high-luminosity  $\mu^+$  and  $\mu^-$  beams. The collider scheme referenced in this paper is based on that of [2]: a high-power proton driver sends a MW beam into a target to produce pions, which are captured and drifted by strong magnetic fields; the pions decay to muons, which are cooled, accelerated, and injected into a collider ring for multi-TeV collisions.

#### FRICTIONAL COOLING

Frictional cooling involves the balancing of energy loss to a moderator with energy gain from an electric field to bring a beam of charged particles to an equilibrium energy and reduce dispersion. It requires the beam be in an energy region where the stopping power of the moderator, the energy loss per unit path length normalized by the medium density,  $(1/\rho) dT/ds$ , increases with increasing kinetic energy. There are two energy regions where this requirement is met (figure 1). Ionization cooling schemes utilize particle beams in the high-energy region [3]. Frictional cooling utilizes particle beams in the low energy region.

Applying an electric field to restore energy loss creates two equilibrium energies: a stable one,  $T_{\rm eq}$ , below the ionization peak of the stopping power curve, where  ${\rm d}T/{\rm d}s \propto \sqrt{T}$ , and an unstable one,  $T'_{\rm eq}$ , above the peak, where  ${\rm d}T/{\rm d}s$  decreases with increasing kinetic energy. Particles with kinetic energies below  $T_{\rm eq}$  and  $T'_{\rm eq}$  decelerate; those with kinetic energies between  $T_{\rm eq}$  and  $T'_{\rm eq}$  decelerate. The

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Figure 1: Stopping power (solid curve) of helium on  $\mu^+$ and the accelerating power of an example restoring electric field (horizontal dashed line) that yields an equilibrium energy  $T_{eq} = 1 \text{ keV}$ .

coolable energy region, in which particles' energies converge on  $T_{\rm eq}$ , is defined by  $T < T'_{\rm eq}$ . Additionally, restoring lost energy only in the longitudinal direction cools the beam in the tranverse directions.

For a chosen equilibrium energy, the electric field strength required to balance the energy loss scales directly with the density of the moderator. To keep the electric field strength within a feasible range, the density of the moderator must be low. Helium and hydrogen gases are good moderators because they have low densities and supress the capture of electrons by the cooled particles [4, 5] and the capture of the particles by the medium atoms [6].

## **COOLING CELL**

In the muon collider scheme described above, a frictional cooling cell, a gas-filled cylindrical chamber, would immediately follow a pion decay channel. The energies of muons exiting the channel are on the order of 100 MeV, well above the coolable energy region. The cooling cell must first decelerate the muons to below  $T'_{\rm eq}$ . An axial magnetic field guides them through the cell as they lose energy to the moderator gas. The force on the muons is

$$\boldsymbol{F} = e\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right) - \frac{\mathrm{d}T}{\mathrm{d}s}\hat{\boldsymbol{v}},$$

where  $\hat{v}$  is a unit vector in the direction of the muon velocity, v. The restoring electric field must be perpendicular to the magnetic field, which is parallel to the beam direction ( $\hat{z}$ ), to ensure minimal energy gain for the muons until E and  $v \times B$  are of comparable magnitude. The strengths of the fields are tuned such that this condition is met when

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Figure 2: Sketch of the frictional cooling cell showing the orientation of the electric and magnetic fields and the location of the reacceleration field.

the muons have energies below  $T'_{eq}$ . This field configuration also limits the maximum kinetic energy of secondary electrons, limiting electric breakdown in the gas [7].

In the presence of the magnetic field in the z direction and electric field in the x direction, muons drift in the y direction. The electric field strength is varied sinusoidally as a function of z to reverse the drift direction periodically, preventing muons from drifting out of the cooling cell (figure 2). Once the muons slow down to the coolable energy region, they quickly come to the equilibrium energy and drift transversely out of the cell. The transverse size of the cooling cell is chosen to reduce loss of muons to decay. At kinetic energies  $T \ll m_{\mu}c^2$ , the distance traveled by a muon before decaying is

$$v\tau_{\mu} \approx \sqrt{T/\text{eV}} \cdot 10 \text{ cm}.$$

With equilibrium energies on the order of keV, the tranverse size of the cooling cell must be on the order of tens of cm.

After exiting the cooling cell, the muons are reacceler-



Figure 3: End of the trajectory of a  $\mu^+$  in the frictional cooling cell in the *x*-*y* plane.

ated in the z direction. At the end of the cooling cell region they enter a rapid reacceleration and bunching section.

## SIMULATION

The results of a first simulation of a frictional cooling cell for use in a muon collider were published in [8]. The cooling cell in the simulation is a cylinder 11 m long and 40 cm in diameter (figure 2). The magnetic field strength is 5 T, and the electric field strength is 5 MV/m oscillated sinusoidally along z with a period of 60 cm.

To minimize the size of the cooling cell, an electric field antiparallel to the beam direction is placed at the end of the cell. It reflects the fastest muons, which transverse the full length of the cell without slowing down to the coolable energy region, back into the cell. The field strength is varied according to

$$E = E_0 \begin{cases} 1 & \text{if } t < t_1, \\ (t - t_1)/(t_2 - t_1) & \text{if } t_1 \le t < t_2, \\ 0 & \text{if } t_2 \le t, \end{cases}$$

where  $E_0 = 5$  MV/m, and  $t_1$  and  $t_2$  have been optimized to 100 ns and 439 ns. The reflection field is constant for a time long enough to reverse the direction of the muons and then linearly falls off to zero, so as not to restore the total original energy of the muons. It therefore provides a phase rotation, increasing the number of muons at lower energies.

Figure 3 shows the end of the trajectory for one  $\mu^+$  in the simulation of the cooling cell. The spiraling in the *x-y* plane and the swim in the *y* direction are both visible. The muon spirals in the magnetic field losing energy to the helium gas until it reaches the coolable energy region and quickly decelerates to the equilibrium energy (figure 4), exiting the cooling cell in the transversely at a Lorentz angle to the electric field direction. The kinks in figure 4 are due to large-angle scattering that sends the muon in a direction opposed to the electric field, after which it must be turned and reaccelerated to the equilibrium energy.



Figure 4: Kinetic energy as a function of time for the end of the trajectory of a  $\mu^+$  in the frictional cooling cell.

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Variable	RMS		
	Initial	Final	
		1st beamlet	2nd beamlet
x (cm)	9.48	4.48	5.30
y (cm)	2.68	4.76	3.85
$\beta ct \ (cm)$	176.93	1600.71	1612.02
$p_x  (\text{MeV}/c)$	20.24	0.16	0.16
$p_y  ({\rm MeV}/c)$	20.39	0.15	0.16
$p_z$ (MeV/c)	37.91	1.18	1.22

Table 1: RMS values for the  $\mu^+$  beam.

Table 2: Normalized emittances for the  $\mu^+$  beams

	Initial	Final	
		1st Beamlet	2nd Beamlet
$\epsilon_{\rm T} \ (\pi {\rm m})^2$	$9.51 \times 10^{-6}$	$4.56\times10^{-10}$	$4.71\times10^{-10}$
$\epsilon_{\rm L}$ ( $\pi {\rm m}$ )	$2.0 \times 10^{-1}$	$5.7 \times 10^{-2}$	$5.9 \times 10^{-2}$
$\epsilon_{6D} \ (\pi m)^3$	$1.92 \times 10^{-6}$	$2.61 \times 10^{-11}$	$2.79 \times 10^{-11}$

A weak electric field (10 kV/m) accelerates the muons in the z direction after they exit the gas cell. At the end of the cell, a time dependent electric field rapidly accelerates the beam. The field,

$$E = E_0 \left( 1 + \frac{t - t_0}{A} + \left( \frac{t - t_0}{B} \right)^2 \right),$$

where  $E_0 = 14 \text{ kV/m}$ , A = 75.7 ns, and B = 45.4 ns, and  $t_0$  is a timing offset, is present for 43.5 m, after which a constant field of 1.13 MV/m is present. The reacceleration increases the momentum spread (to 5 MeV at a mean energy of 150 MeV) while decreasing the time spread of the beam from 1 µs to 3 ns.

## Emittance & Yield

The time required for the fastest coolable muons to reach the equilibrium energy in one pass through the cell is approximately 300 ns. The time to drift out of the gas cell transversely is on average approximately 200 ns. Incorporating the additional time for the fastest muons to make a first pass through the cell, be reflected back in, and be cooled, the maximum time for cooling a muon is on the order of 1  $\mu$ s. The final yield of cooled muons is  $2.1 \times 10^{-3} \mu^+$  per proton incident on the pion production target.

The initial and final parameters of the beam are listed in table 1. Since the direction of the electric field in the cooling cell is periodically reversed, muons are extracted in both the positive and negative y directions, yielding two beams which must be merged.

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The final emittance of the two beams is listed in table 2. An emittance reduction of 5 orders of magnitude is achieved.

## OUTLOOK

The results of the first simulations of a frictional cooling scheme for a muon collider are promising. A new simulation tool called CoolSim [9] has been developed at the Max Planck Institute for Physics in Munich, Germany, to improve upon the previous simulation. CoolSim is based on Geant4 [10] and implements new low-energy physics processes into the Geant4 framework. Most importantly, it models charge exchange processes for  $\mu^+$  and protons, which is missing from both the Geant4 framework and the previous frictional cooling simulations. Inclusion of these processes will allow us to refine the optimal field configurations for a frictional cooling cell.

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