DYNAMIC APERTURE LIMIT CAUSED BY IR NONLINEARITIES IN EXTREMELY LOW BETA B FACTORIES

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Abstract

Super B factories are designed with an extremely low beta (β_y~200μm) at the interaction point. Nonlinearity existing in the interaction region is strong. While the crab waist scheme requires few nonlinearity between two strong sextupoles located at the both side of the interaction point. This collision scheme, low beta and/or crab waist, is very attractive for the beam-beam performance. However the dynamic aperture is very small in several trials of simulations in KEKB and SuperKEKB. We study why the dynamic aperture is so small using simple models.

INTRODUCTION

We discuss dynamic aperture using simple models consists of the interaction region (IR) and residual section denoted "arc". The arc section is represented by linear transformation: that is, all nonlinearity in arc section is assumed cancelled perfectly. Nonlinearity of kinematic term and quadrupole fringe in IR section is main issue in this paper.

Kinematic nonlinearity is dominant in the region of high $\gamma=(1+\alpha^2)/\beta$. Figure 1 shows γ along the ring for SuperKEKB. The high γ area is limited in several meters of IR section.

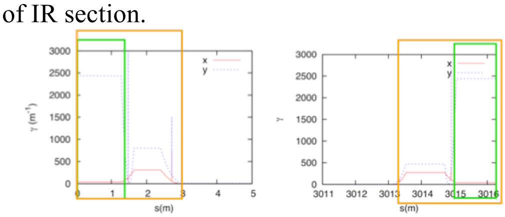

Figure 1: γ function in SuperKEKB. Green area is inside of the first quadrupole magnets, and Yellow area contains sencond quadrupole magnets.

MODEL CONSISTS OF IR DRIFT SPACE AND ARC

Hamiltonian in drift space for relativistic particles is expressed by

$$H=(1+\delta)-\sqrt{(1+\delta)^2-p_x^2-p_y^2} \qquad (1)$$

We first consider linear transformation for arc section, which is outside of the first quadrupole faces of the interaction region, and the drift space of the interaction region as shown in Figure 2.

The linear transformation of the outside M_{arc} related to the revolution matrix as follows,

$$M_{-1,0}^{-1}M_{arc}M_{1,0}=M_0 \qquad (2)$$

where $M_{+-,0}$ is the transfer matrix from the interaction point to the first quadrupole of downstream (+) or upstream (-).

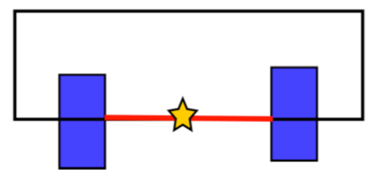

Figure 2: Simplified model consists of IR(red) drift and arc (black) for the dynamic aperture evaluation.

This Hamiltonian contains nonlinear terms. They (H_n) are extracted by subtracting the linear part as follows,

$$\begin{aligned} H_n &= (1+\delta)-\sqrt{(1+\delta)^2-p_x^2-p_y^2}-\frac{p_x^2+p_y^2}{2} \\ &= -\frac{(p_x^2+p_y^2)\delta}{2(1+\delta)}+\frac{1}{8}\frac{(p_x^2+p_y^2)^2}{(1+\delta)^3}+\frac{1}{16}\frac{(p_x^2+p_y^2)^4}{(1+\delta)^5}+\ldots \end{aligned} \qquad (3)$$

The term to extract, $(p_x^2+p_y^2)/2$, corresponds to the linear transformation $M_{+-,0}$, which gives $x_{+-}=x_0+p_xL_0$, where L_0 is the distance of quadrupole face to IP.

The first term of the 2-nd line, which is defined by H_ξ, corresponds to the chromaticity,

$$H_\xi=-\frac{(p_x^2+p_y^2)\delta}{2(1+\delta)} \qquad (4)$$

The Hamiltonian H_ξ gives the transformation

$$\bar{x}=x-p_x\frac{L_0\delta}{1+\delta}\equiv x+p_xL_\delta \quad , \quad L_\delta=-\frac{L_0\delta}{1+\delta} \qquad (5)$$

Actually the transformation multiplying the transverse revolution matrix gives

$$\begin{pmatrix} 1 & L_\delta \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu \end{pmatrix}\begin{pmatrix} 1 & L_\delta \\ 0 & 1 \end{pmatrix} \qquad (6)$$

$$=\begin{pmatrix} \cos\mu-\frac{L_\delta\sin\mu}{\beta} & \left(\beta-\frac{L_\delta^2}{\beta}\right)\sin\mu+2L_\delta\cos\mu \\ -\frac{\sin\mu}{\beta} & \cos\mu-\frac{L_\delta\sin\mu}{\beta} \end{pmatrix}$$

The momentum $p_{x,y}$, which is a constant in the drift space, is expressed by $\gamma_{x,y}J_{x,y}$ near the interaction point.

In this picture the chromaticity, which is expressed by the integral of the γ, is equal to the integral of $K\beta$ in ordinary expression,

$$4\pi\xi=\oint\gamma ds=\oint K\beta ds \qquad (7)$$

The revolution matrix is given by multiplication of linear transformation of all accelerator elements. The one turn map is expressed by the revolution map and the nonlinear map extracted from the Hamiltonian in Eq.(3) as follows,

$$e^{-:H_K:L_0}M_0e^{-:H_K:L_0} \qquad (8)$$

The quadratic term for the transverse coordinate is eliminated by a perfect chromaticity correction.

$$
\begin{aligned}
H_K &= (1+\delta) - \sqrt{(1+\delta)^2 - p_x^2 - p_y^2} - \frac{(p_x^2+p_y^2)\delta}{2(1-\delta)} \\
&= \frac{1}{8}\frac{(p_x^2+p_y^2)^2}{(1+\delta)^3} + \frac{1}{16}\frac{(p_x^2+p_y^2)^4}{(1+\delta)^5} + \ldots
\end{aligned} \tag{9}
$$

The residual terms are geometric nonlinear terms. The lowest order perturbation, is expressed by

$$2L_0 H_K \approx \frac{p_y^4}{4} L_0 = \frac{J_y^2}{\beta_{y,0}^2} L_0 \tag{10}$$

The amplitude dependent tune shift is given by

$$\Delta\nu_y = \frac{1}{2\pi} L_0 \frac{J_y}{\beta_{y,0}^2} = \frac{J_y}{2\times 10^{-6}}. \tag{11}$$

Small beta results small dynamic aperture. The evaluated number is for SuperKEKB.

EDGE NONLINEARITY OF QUADRUPOLE MAGNETS

The nonlinear field of quadrupole magnets are expressed by [1]

$$H_E = -\frac{k}{1+\delta} \frac{y p_y (3x^2+y^2) - x p_x (3y^2+x^2)}{12} \tag{12}$$

Transverse coordinate at quadrupole magnets are roughly determined by p_x, p_y, $x_{+-}=x_0+p_xL_0\sim p_xL_0$, $y_{+-}=y_0+p_yL_0\sim p_yL_0$.

$$e^{:H_0:} e^{-:H_E:} e^{-:H_0:} = e^{-:H_{E,0}} \tag{13}$$

The edge effect is expressed using the coordinate at IP by

$$H_{E,0} \approx -\frac{kL_0^3}{1+\delta}\frac{p_y^4 - p_x^4}{12} \approx -\frac{kL_0^3}{1+\delta}\left(\frac{J_y^2}{12\beta_{y,0}^2} - \frac{J_x^2}{12\beta_{x,0}^2}\right) \tag{14}$$

$$\Delta\nu_y \approx -\frac{kL_0^3}{12\pi(1+\delta)}\frac{J_y}{\beta_{y,0}^2} \approx \frac{J_y}{4\times 10^{-6}}$$

Second quadrupole magnet is dominant for the horizontal tune shift.

CRAB WAIST SCHEME

In the crab waist scheme, two sextupoles are installed so as to give an effective Hamiltonian, xp_y^2/θ at IP, where θ is the full crossing angle. Actually the sextupole magnet, which has xy^2 component, is installed at positions where the horizontal betatron phase difference $n\pi$ and the vertical difference $(n+1/2)\pi$ with the strength considering the beta function amplitude. The one turn map including the crab waist sextupole is expressed by

$$e^{-:H_K:L_0} e^{-:xp_y^2:/\theta} M_0 e^{:xp_y^2:/\theta} e^{-:H_K:L_0} \tag{15}$$

Changing the order

$$M_0 e^{:xp_y^2:/\theta} e^{-:H_K:L_0} e^{-:H_K:L0} e^{-:xp_y^2:/\theta} \tag{16}$$

When H_K=0, the crab waist nonlinearity cancel and then the transformation is linear. The crab waist sextupole does not affect the dynamic aperture. We have to make effort to eliminate the nonlineairy H_K inside of the carb waist sextupole pair.

A MODEL CONTAINING FOUR QUADRUPOLE MAGNETS

The first quadrupole magnets defocus the horizontal beta function. Contributions of the second quadrupole magnets are also important for the nonlinearity, especially in horizontal.

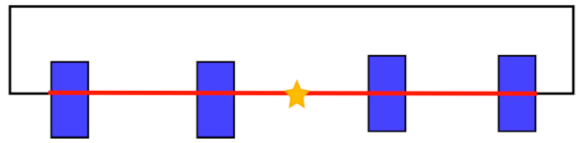

Figure 3: A model consists of IR section (red) and arc (black).

We discuss a little complex model, which consists of four quadrupole magnets as shown in Figure 3. The transfer map of the IR region is expressed by

$$\mathcal{M}_{IR} = e^{-H_{QF}} e^{-H_{L1}} e^{-H_{QD}} e^{-H_{L0}} e^{-H_{L0}} e^{-H_{QD}} e^{-H_{L1}} e^{-H_{QF}} \tag{17}$$

The one turn map is multiplication of maps of IR and arc. The transfer map of arc is assumed linear.

$$\mathcal{M}_{rev} = \mathcal{M}_{IR}\mathcal{M}_{arc} \tag{18}$$

The relation of arc transfer matrix and revolution matrix is given by

$$\mathcal{M}_{arc} = M_{arc} = M_{IR}^{-1} M_0 \tag{19}$$

We subtract the linear transformation from the IR map,

$$\mathcal{M}'_{IR} = \mathcal{M}_{IR} M_{IR}^{-1} \tag{20}$$

Then one turn map is expressed by the revolution matrix and nonlinear transformation in IR.

$$\mathcal{M}_{rev} = \mathcal{M}'_{IR} M_0 \tag{21}$$

On momentum aperture is evaluated by this map.

Figure 4 shows the transverse dynamic aperture for KEKB. The IR parameters, β_x=0.6m β_y=6mm L_0=1.3m, L_{QD}=2.3m K_{QD}=-0.69m^{-1}, L_1=2.57m, L_{QF}=2.07m, K_{QF}=0.24m^{-1}. The aperture is quite large without crab waist, but shrank remarkably with crab waist. This behaviour has been seen using SAD to install crab waist scheme in KEKB.

Figure 5 shows the transverse dynamic aperture for superKEKB (top) and SuperB(bottom). The IR parameters for SuperKEKB are L_0=0.73m, L_{QD}=0.39m K_{QD}=-1.7m^{-1}, L_1=0.69m, L_{QF}=0.35m, K_{QF}=0.83m^{-1}, β_x=2cm β_y=0.2mm. They for SuperB are L_0=0.4m,L_{QD}=0.45m K_{QD}=-2.7m^{-1}, L_1=0.4m, L_{QF}=0.20m, K_{QF}=1.2m^{-1}, β_x=2cm β_y=0.2mm. Big difference between SuperKEKB and SuperB is the size of IR. IR of SuperB is more compact than that of SuperKEKB.

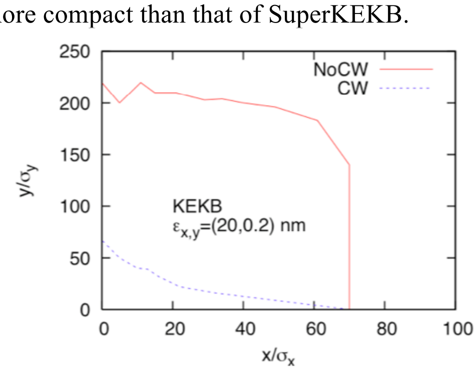

Figure 4: Dynamic aperture for the model of KEKB with and without crab waist sextupoles.

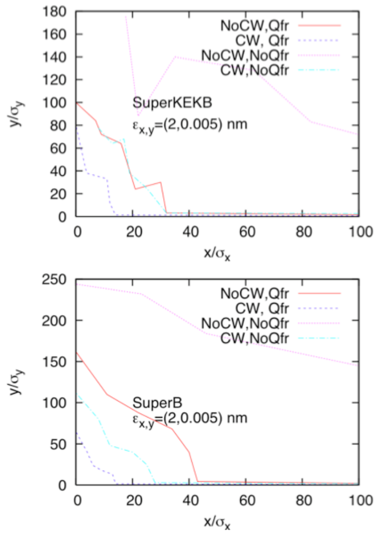

Figure 5: Dynamic aperture for the models of SuperKEKB(top) and SuperB(bottom) with and without crab waist sextupoles.

CHROMATICITY CORRECTION

The transfer map of IR is expressed by

$$\mathcal{M}'_{IR} = \prod_i^{IR} e^{-H_i} = \prod_i e^{H_{2,i}} e^{-H_{\xi,i}} e^{-H_{K,i}} e^{-H_{2,i}} \tag{22}$$

Eliminating the chromatic term H_ξ, achromatic map is obtained as

$$\mathcal{M}''_{IR} = \prod_i e^{H_{2,i}} e^{-H_{K,i}} e^{-H_{2,i}} \tag{23}$$

The one turn map consists of $\mathcal{M}$'' and M_0 gives the dynamic aperture independent of momentum deviation; no aperture degradation for a momentum deviation. This situation may be trivial in this model and is unphysical.

We next consider the chromaticity is corrected outside of the IR. Transfer map, which contains quadratic term of transverse coordinates, is multiplied in the both side of IR. The quadratic term is given by Eq.(4). This map gives a possible perfect chromaticity correction.

$$e^{-H_{c,out}} \mathcal{M}'_{IR} e^{-H_{c,in}} \tag{24}$$

where

$$e^{H_{c,in(out)}} = \prod_i^{up(down)stream} e^{H_{2,i}} e^{-H_{\xi,i}} e^{-H_{2,i}} \tag{25}$$

Figure 6 shows the off-momentum dynamic aperture after the chromaticity correction. In the aperture survey, particles are initialized with horizontal, longitudinal amplitude and small vertical amplitude of 1% of the horizontal in the emittance unit. The dynamic aperture with crab waist is also very small.

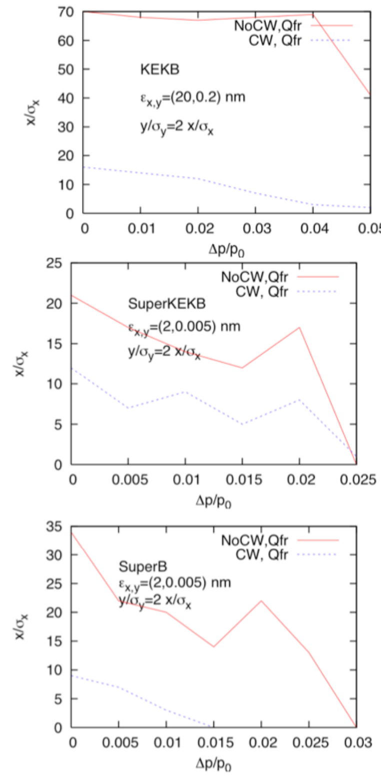

Figure 6: Off momentum dynamic aperture for the models of KEKB(top), SuperKEKB(centre) and SuperB(bottom).

SUMMARY

Dynamic aperture is estimated using IR model including only 4 quadrupole magnets. Effects of Kinematic nonlinearity and quadrupole edge nonlinearity on the dynamic aperture are investigated. It seems to be very serious especially for crab waist scheme.

These results do not mean a fundamental limit of the dynamic aperture. Insertions of nonlinear magnets may recover the aperture. In SuperB factoris designed in LNF-Frascati, the dynamic aperture with crab waist sextupoles do not seem to be serious [2]. We need detailed tuning and studies for the dynamic aperture in collaboration with LNF and BINP teams.

REFERENCES

[1] E. Forest and J. Milutinovic, Nucl. Inst. and Methods in Phys. Res. A269 , 478 (1988).

[2] E. Levichev et al., private communications.