# TRANSIENT BEAM LOADING COMPENSATION AT RF ACCELERATION OF INTENSE SHORT-PULSED ELECTRON BEAMS

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*Abstract*

Acceleration of intensive electron beams in transient mode with energy spread less than 1% is the current problem for RF linacs. The transient beam loading phenomenon, consisting in coherent radiation of sequence of charged bunches, results in time dependence of electron energy loss within a beam pulse. In this work the method of delay of a beam relative to an RF pulse for energy compensation at accelerating intense short-pulsed electron beams is discussed. The efficiency of given method versus the dispersion of group velocity, phase advance per cell of an RF structure and envelope of input RF field is studied.

## INTRODUCTION

Transient beam loading is one of key problems which restrict obtaining a small energy spread along a bunch train in high-intensity RF electron linacs. Since 1960th and to the present time, different methods of beam loading compensation have been developed. There are several basic techniques, namely ΔT, ΔS, ΔF, and ΔA schemes. The ΔT scheme consists in beam injection before an RF pulse fills a section [1,2]. The method [3,4] where an RF waveform is linearly ramped during the filling time of a structure is classified in [5] as the ΔS scheme. Additional RF structures, operating at slightly different frequencies (ΔF scheme), have been proposed by authors [6] to compensate the beam voltage. They have shown that the combination of the ΔT+ΔS+ΔF techniques can save over 30% of RF power. It should be noted, that the techniques mentioned above, as a rule, were developed for the SLED driven constant-gradient accelerator structures. The idea of the ΔA method [5] consists in compensation of the field reduction due to beam loading by an appropriate steps of the amplitude of an RF power pulse.

The ΔT scheme is the simplest technique, which has the advantage when bunch train length shorter than the filling time notably [2]. That can be short-pulsed RF linac-injectors for ring accelerators with moderate energies. Below we will study characteristics of the ΔT scheme used for a single constant–impedance (CZ) accelerating structure taking into account finite width of rise time of an input RF waveform. We will also consider two ways of application of the ΔT scheme, namely: 1) a beam is switched on/off before a pulse from an RF source reaches the end of the structure; 2) the beam is injected when the wave-front of the RF pulse is crossing the end cell.

## COMPENSATION CHARACTERISTICS

Starting from the equations of excitation of waveguides and using Laplace transformation, the time dependent longitudinal electric field over a CZ section can be derived in the general form

$$\begin{aligned} E(t,z) &= \int_0^t E_0(t-t')G(t',z)\,dt' \\ &-\frac{\omega R_{sh}}{2Q}\left[q_{eff}(t)-\int_0^t q_{eff}(t-t')G(t',z)\,dt'\right], \end{aligned} \quad (1)$$

where $t$ is time, z is a longitudinal coordinate; $E_0(t)$ is the envelope of an input RF pulse, $\omega$ is the circular operating frequency, $Q$ is the quality of cells, $G(t,z)$ is the delta-function response of the waveguide [1], $R_{sh}$ is the shunt impedance, $q_{eff}$ is the effective charge of the beam, determined as $q_{eff}(t)=e^{-\frac{\omega}{2Q}t}\int_0^t I(t')e^{\frac{\omega}{2Q}t'}dt'$, $I(t)$ is the beam current averaged over an RF period.

### *Beam Switched On/Off Before Filling of a Section*

We begin with neglecting dispersive effects. In this case $G(t,z)=\delta(t-z/\mathrm{v}_g)$, where $\mathrm{v}_g$ is the group velocity. Let an accelerating section of length $L$ be fed at its upstream end by an input RF waveform with linear rise time $\Delta t_{rise}$ that travels to the other end without reflection (see Fig.1).

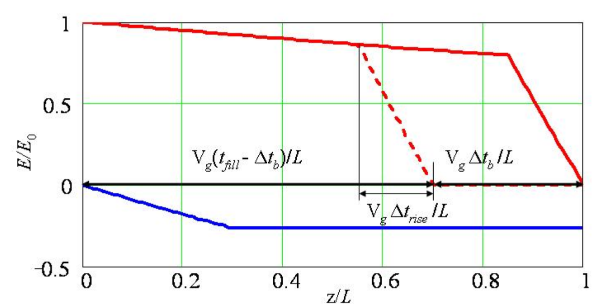


Figure 1: Snapshot RF waveforms of fields from both an RF source (red lines) and a beam (blue line) at the time of the beam switching on/off (dashed/solid lines).

A beam pulse with duration $\Delta t_b$ and zero time of rise and fall is switched on before complete fill of the section at the time $t_0=t_{fill}-\Delta t_b$ (where $t_{fill}$ is the filling time). Also the blue line in Fig.1 illustrates the field induced by beam at time $t_{fill}$ when the beam is switched off.

After integrating Eq.(1) over length $L$, we can find the condition of beam loading compensation, under which a linear time dependence of the energy gain disappears

$$\frac{\Delta t_b}{2t_{fill}}=1-\frac{1}{I}\sqrt{\frac{2PL}{\tau R_{sh}}}e^{-\tau\left(1-\frac{\Delta t_b}{t_{fill}}-\frac{\Delta t_{rise}}{2t_{fill}}\right)}, \quad (2)$$

where $\tau = \omega t_{fill}/2Q$ is the attenuation factor, $R_{sh}$ is the shunt impedance, $P$ is the input RF power. In this case the energy gain has a near-parabolic time profile, and the small energy spread along the bunch train is attained. The leading and trailing bunches have the same energy and the central bunches have a lower energy, see Fig.2.

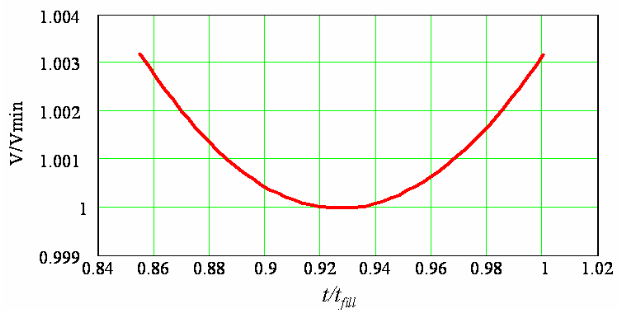


Figure 2: A normalized loaded accelerating voltage as a function of time with beam compensation.

In this situation, RMS energy spread along a bunch train can be derived in the form

$$\sigma = \frac{1}{12\sqrt{5}} \frac{(\tau\nu)^2 e^{-\frac{\tau\nu}{2}}}{\tau\left(1-\frac{\nu}{2}\right)\left(e^{\tau\left(1-\nu-\frac{\varepsilon}{2}\right)}-1\right)+1-\frac{\tau\nu}{2}-e^{-\frac{\tau\nu}{2}}+\frac{(\tau\nu)^2}{24}e^{-\frac{\tau\nu}{2}}}, \quad (3)$$

where $\nu = \Delta t_b / t_{fill}$, $\varepsilon = \Delta t_{rise}/t_{fill}$. The $\nu$-dependences of the RMS energy spread at $\tau$=0.5 are plotted in Fig.3.

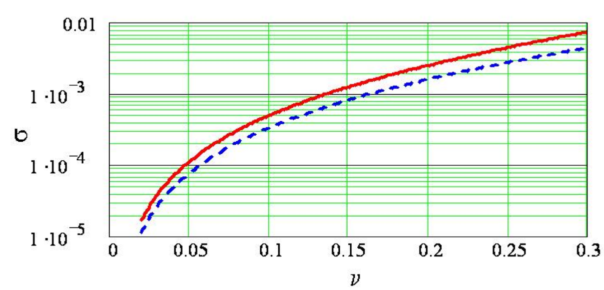


Figure 3: RMS energy spread as a function of $\nu$ at ε=0.5 (solid red line), and ε=0 (dashed blue line), respectively.

The RMS energy spread is weakly dependent on both the attenuation factor and wave-front width. To calculate the optimal input RF power and shunt impedance for given required parameters such as σ, $I$, $L$ and $\bar{V}$ (average loaded voltage), the useful relations can be obtained

$$R_{sh} = 12\sqrt{5}\frac{\bar{V}}{IL}\frac{\sigma}{\tau\nu^2}e^{\frac{\tau\nu}{2}}, \quad P = \frac{3}{2}\sqrt{5}I\bar{V}\sigma\left(\frac{2}{\nu}-1\right)e^{2\tau\left(1-\frac{3\nu}{4}-\frac{\varepsilon}{2}\right)}. \quad (4)$$

### *Beam Switched On/Off During Filling a Section*

Here we consider a somewhat different way of utilization of the ΔT scheme. Let a beam be injected when the linear rising wave-front crosses the end cell of a traveling wave CZ-structure, as shown in Fig.4.

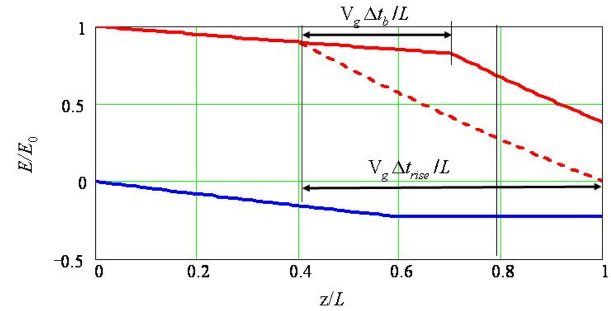


Figure 4: Snapshot RF waveforms of fields from both an RF source (red lines) and a beam (blue line) after the wave-front reaches the end cell. The beam switching on/off corresponds to the dashed/solid lines.

In this case the condition under which the linear time dependence of energy gain can be neglected has form

$$\frac{t_0}{t_{fill}} = -\frac{\Delta t_b}{2t_{fill}}+1+\frac{\Delta t_{rise}}{t_{fill}}+\frac{1}{\tau}\left(1-\frac{\Delta t_{rise}}{t_{fill}}\left(\tau+\eta(t_0)\right)\right)$$
$$\times\left[1-\sqrt{1-2\eta(t_0)\left(\frac{t_0}{t_{fill}}+\frac{\Delta t_b}{2t_{fill}}-\frac{\Delta t_{rise}}{t_{fill}}\right)\Big/\left(1-\frac{\Delta t_{rise}}{t_{fill}}\left(\tau+\eta(t_0)\right)\right)^2}\,\right] \quad (5)$$

with $\eta(t_0) = I\sqrt{\frac{\tau R_{sh} L}{2P}}e^{-\tau\left(\frac{\Delta t_{rise}}{t_{fill}}-\frac{t_0}{t_{fill}}-\frac{\Delta t_b}{2t_{fill}}\right)}$, where $t_0$ is the time of the beam switching on, which ranges from $t_{fill}$ to $t_{fill}+\Delta t_{rise}-\Delta t_b$. The energy gain has a convex upwards near-parabolic time profile (Fig.5).

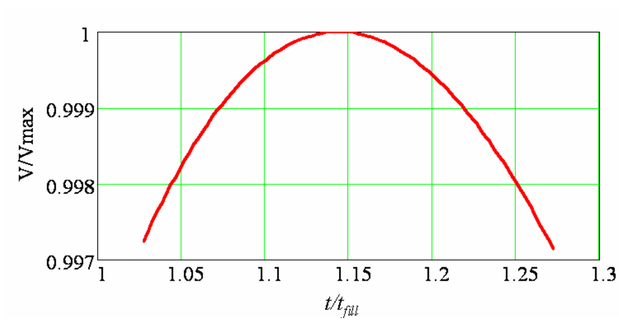


Figure 5: A normalized loaded accelerating voltage as a function of time with beam compensation.

## IMPACT OF DISPERSIVE EFFECTS

Since an RF waveform is distorted upon propagating through an accelerating structure due to dispersive effects, that can result in decreasing effectiveness of the used compensation techniques. To take into account the dispersion, we can use in Eq.(1) a good approximation of the delta-function response for the disk loaded waveguides $G(t,n) = nJ_n(\omega_c t)exp\left[i(n\pi/2-\omega_m t)-\omega_m t/2Q\right]$ [1] ($n$ is a cell number, $\omega_c$ is half width of pass band, $\omega_m$ is the midband circular frequency). Consider RF pulses with initially linear rise time $\Delta t_{rise}$ (50 and 100 ns), which propagate through accelerating structures with differint phase advances per cell ($2\pi/3$ and $\pi/2$) at the operating frequency 2856 MHz but have the same the group velocity of 4% of the speed of light and the filling time $t_{fill}$=292 ns. Fig.6 shows snapshots of the normalized RF waveforms in the $2\pi/3$ structures with the turned off/on dispersive effects (dot/solid line). The unloaded accelerating voltage is the integral of these RF

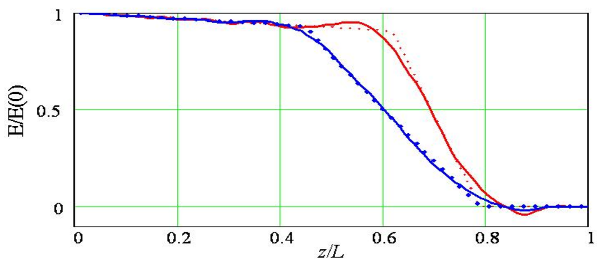


Figure 6: Snapshot RF waveforms in $2\pi/3$ CZ structure for $\Delta t_{rise}$ =50 ns (red line) and $\Delta t_{rise}$ =100 ns (blue line).

waveforms. The Fig.7 illustrates the time dependences of a relative difference $\Delta$ between unloaded accelerating voltage obtained with the turned off/on dispersive effects.

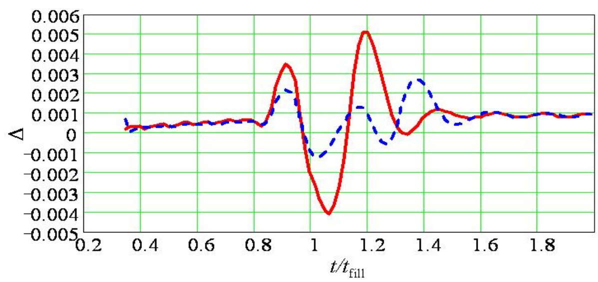


Figure 7: $\Delta$ as function of time for two RF pulses propagating in the $2\pi/3$ CZ structure with rise times 50 ns (red solid line) and 100 ns (blue dashed line).

One can see, that for the injection time up to 0.8 $t_{fill}$ the contribution of the dispersive effects to the beam energy spread is insignificant, less than $10^{-3}$. After this limit, the phase and amplitude modulation of the wave-front, due to the dispersion, strongly impacts on the energy spread, which, however, can be suppressed by lengthening the rise time (blue dashed line). To exclude contribution of the phase modulation to the energy spread, the structure with $\pi/2$ phase advance per cell may be chosen. The corresponding snapshot RF waveforms are plotted in Fig.8 for the $\pi/2$ CZ structure. Fig.9 demonstrates the significant reduction of influence of dispersive effects on the beam energy spread in the $\pi/2$ structure. Contribution of the dispersion to the energy spread amounts to of $10^{-4}$ by order of magnitude for the injection time up to 0.9 $t_{fill}$. If the injection time is greater than this value, the wave-front modulation also insignificantly impacts on the energy spread in the case when the rise time is 100 ns.

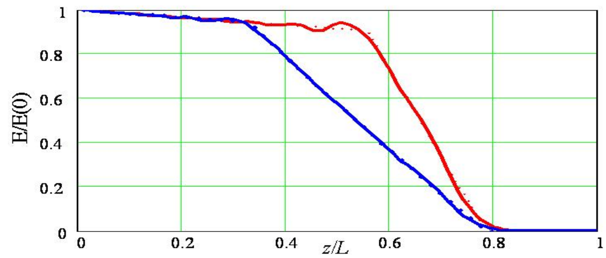


Figure 8: Snapshot RF waveforms in the $\pi/2$ CZ structure. for $\Delta t_{rise}$ =50 ns (red line) and $\Delta t_{rise}$ =100 ns (blue line).

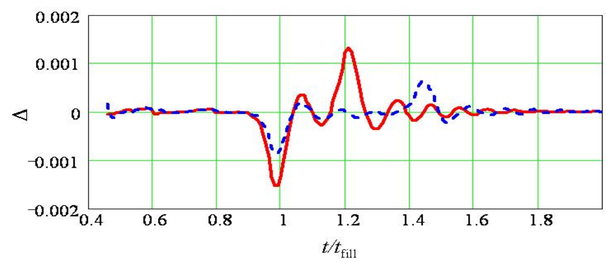


Figure 9: $\Delta$ as function of time for two RF pulses with rise times 50 ns (red solid line) and 100 ns (blue dashed line) in the $\pi/2$ CZ structure.

## SUMMARY

The spatiotemporal dependence of an RF field in the CZ section loaded by a beam has been derived in the general form. The features of two beam loading compensation techniques taking into account a finite rise time of an RF waveform have been considered. The first one (a beam is switched on/off before an RF pulse reaches the end cell) is little sensitive with respect to dispersive effects and can provide the RMS energy spread of about 0.1% at the beam duration less than 0.15 $t_{fill}$, although the finite rise time reduces the acceleration efficiency. The second (a beam is switched on/off during an wave-front is crossing the end cell) has the greater efficiency but is more sensitive to the dispersive effects. To suppress the impact of dispersion, the $\pi/2$ structure and lengthening the rise time are proposed.

## REFERENCES

[1] J. Leiss, in Linear Accelerator edt. by P.Lapostolle and A.Septier, Amsterdam, Netherlands: (North-Holland, 1970).
[2] J. W. Wang "Rf Properties Of Periodic Accelerating Structures For Linear SLAC" – 339, UC – 28, (A)
[3] K. A. Thompson and R. D. Ruth, "Simulation and Compensation of Multibunch Energy Variation in NLC", PAC 1993, p. 3693; http://www.JACoW.org.
[4] C. Adolphsen et al., SLAC-PUB-7518 (May 1997).
[5] W. Mondelaers et al., "Transient Beam Loading Compensation in Low-Energy Pulsed Linear Electron Accelerators" LINAC'00, Monterey, Aug 2000, MOE08, p. 284 (2000); http://www.JACoW.org
[6] Z. Li et al., SLAC-PUB-7429 (March 1997)