# LIGHT SOURCE BASED ON MULTITURN-CIRCULATION OF BEAM OF ENERGY RECOVERY LINAC 

T. Nakamura, JASRI/SPring-8, 1-1-1 Kouto, Sayo-cho, Hyogo 679-5198, Japan

## Abstract

Light sources based energy recovery linacs (ERL) are ones of the candidates for future light sources. We proposed a method of multiturn circulation of an ERL beam in a light source to increase the circulation current at a light source several times more than the beam current from ERL. The method employs RF deflection cavities and the method to calculate the parameters of RF deflection, several possible scheme of its application, and some calculation on a model light source are reported in this report.

## MULTITURN CIRCULATION ERL LIGHT SOURCE

The multiturn circulation method of a beam from a energy recovery linac (ERL) [1-3] can increase the average current by the increase the bunch rate of a light source, by a factor of the number of turns circulating in the light source. The high average current of an electron source is one of the most serious limiting factors. And beam instabilities driven by high average current in accelerator tubes in an ERL make it difficult to realize a multi-pass ERL in which the beam passes through accelerator tubes for acceleration/deceleration several times, to save constriction and operation cost. This multiturn method can reduce the average current of a electron source and an ERL, therefore, this method is a candidate of a method to solve the average current related problems of ERLs.

## N-PASS SYSTEM

We will fist consider a system which kicks every N bunches, not kicking other bunches. We call this system as N -switch system. The system is composed of magnets and several transverse deflection cavities, and produces a time dependent transverse kick. We define this kick and its time derivative as $\theta(t)$ and $\theta^{\prime}(t)=d \theta(t) / d t$, respectively. We assume that the bunch spacing $T_{b}$ is constant for all bunches and the kick $\theta(t)$ is periodic with the period $T_{D}=N T_{b}$. Then, the timing of the passage of the n -th bunch is

$$
\begin{equation*}
t_{n}=T_{b} n=\frac{T_{D}}{N} n, n=0,1,2, \ldots, N-1, \tag{1}
\end{equation*}
$$

and the required condition on the kick of the system is

$$
\begin{align*}
& \theta\left(t_{n}\right)= \begin{cases}\theta_{0} & (n=0) \\
0 & (n \neq 0)\end{cases}  \tag{2}\\
& \theta^{\prime}\left(t_{n}\right)=0 \tag{3}
\end{align*}
$$

where the second one is necessary not to induce the crab motion of bunches. We take its Fourier cosine series with $N$ frequencies including DC as

$$
\begin{align*}
& \theta(t)=\theta_{0} \sum_{k=0}^{N-1} a_{k} \cos k \omega_{D} t  \tag{4}\\
& \theta^{\prime}(t)=-\theta_{0} \omega_{D} \sum_{k=0}^{N-1} a_{k} k \sin k \omega_{D} t \tag{5}
\end{align*}
$$

where $\omega_{D}=2 \pi / T_{D}$. Then we have the $2 N$ equations for the condition Eq. (2) and (3) as

$$
\begin{align*}
& \sum_{k=0}^{N-1} a_{k} \cos \frac{2 \pi}{N} n k= \begin{cases}1 & (n=0) \\
0 & (n \neq 0)\end{cases}  \tag{6}\\
& \sum_{k=0}^{N-1} a_{k} k \sin \frac{2 \pi}{N} n k=0 \tag{7}
\end{align*}
$$

for $n=0,1,2, \ldots, N-1$. However, the kick has mirror symmetry for $t=T_{D} / 2$, therefore only the N independent condition is left and we have a solution $\left\{a_{k}: k=0,1,2, \ldots\right.$, $N-1\}$ of Eq.(6) and (7). The kick by N-switch systems for several $N$ are shown in Fig. 1 .


Figure 1: Examples of the kick by N -switch systems for $N=2,3,4,5$ and 6 . The bunch at the timing $n=0$ is kicked by this system and other bunches ( $n \neq 0$ ) are not kicked.

For example, the two-switch system has a kick:

$$
\theta\left(t_{n}\right)=\theta_{0} \frac{1}{2}\left(1+\cos \frac{2 \pi}{2} n\right)
$$

as shown in Fig. 2 and a three-switch system has a kick:

$$
\theta\left(t_{n}\right)=\theta_{0} \frac{1}{9}\left(3+4 \cos \frac{2 \pi}{3} n+2 \cos \frac{2 \pi}{3} 2 n\right)
$$

as shown also in Fig. 2.
The kick of each frequency component is produced by each cavity or magnet. Therefore, those components have to be placed for the bunch to form a chicane like orbit of the bunch center of mass motion and the crab motion. Some examples of systems are shown in Fig. 3.
This system can be used for the multiturn circulation scheme as shown in Fig. 5. The N-switch system is used to injection and extraction of every N bunches to/from the ring, and other bunches just pass through the system and start the next turn. The circulation period of the ring is adjusted for the passage timing of a bunch in the N -switch system to shift $n$ by one after one turn of the ring.

The required kick voltage to kick the beam by a few micro radians is a few MV which is produced by current deflection cavities in the world [4]. The angular spread by the time dependent kick on the finite length bunch is small compared with the angular spread of the beam.


Figure 2: Kick of each frequency components (DC, $f_{D}$, $2 f_{D}$ ) of two-switch (top) and three-switch system (bottom).


Figure 3: The configuration of two-switch system (left) and three-switch system (right). Bunches passing the system without total kick form a chicane like orbit.


Figure 4: Possible application of N -switch system for multiturn circulation. Left: the number of turns is $N$ with single N -switch system. Right: the number of turns is $N_{1} \times N_{2}$ with the cascade of $N_{1}$-switch and $N_{2}$-switch systems.

## RADIATION EXCITATION

The one of the most serious problems on the multiturn scheme is the degradation of the beam quality by radiation excitation during the circulation. The growth of the emittance in a ring is estimated with the parameters of an ERL and the SPring-8 storage ring listed in Table 1, in which the definition of the symbols for the parameters are also shown.

The equation of the growth of the emittance $\varepsilon$ [5] is

$$
\frac{d \varepsilon}{d t}=-\frac{2}{\tau_{\beta}}\left(\varepsilon-\varepsilon_{\text {Ring }}\right),
$$

then, the emittance in the n -th turn is

$$
\varepsilon(n)=\varepsilon_{E R L}+n \frac{2 T_{0}}{\tau_{\beta}} \varepsilon_{\text {Ring }}
$$

The factor of the increase $\left(2 T_{0} / \tau_{\beta}\right) \varepsilon_{\text {Ring }}$ is proportional to $E^{5}$.


Figure 5: Nine-turn circulation system of beam from an ERL in a light source ring. The configuration is different from those shown in Fig. 5. A bunch which comes from ERL passes the beam line and the ring following the numbers $0 \rightarrow 1$ $\rightarrow 2 \rightarrow, \ldots, \rightarrow 12$, then goes back to ERL. The case with three-pass ERL is shown here.

For the energy spread, the period during the multiturn circulation is much less than synchrotron oscillation period. This is not the case of usual storage rings that the discussion in Ref. [5] is focused on, thus, we need to modify the result in Ref. [5] and have

$$
\sigma_{\delta}(n)^{2}=\sigma_{\delta, E R L}^{2}+n \frac{4 T_{0}}{\tau_{E}} \sigma_{\delta, R i n g}^{2}
$$

where we assume $\sigma_{\delta}(n) \ll \sigma_{\delta, \text { Ring }}$.
Table I: Ring and ERL parameters

| Storage ring with achromat lattice |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Energy | $E$ | 8 | 6 | GeV |  |
| Revolution period | $T_{0}$ | 4.8 |  | $\mu \mathrm{~s}$ |  |
| Longitudinal damping time | $\tau_{s}$ | 4.2 | 9.8 | ms |  |
| Transverse damping time | $\tau_{\beta}$ | 8.3 | 19.7 | ms |  |
| Natural emittance | $\varepsilon_{\text {Ring }}$ | 6 | 3.4 | nmrad |  |
| Natural energy spread (rms) | $\sigma_{\delta, R i n g}$ | 0.11 | 0.081 | $\%$ |  |
| ERL beam |  |  |  |  |  |
| Emittance | $\varepsilon_{E R L}$ | 19 | 26 | pm rad |  |
| Normalized emittance | 0.3 |  |  |  |  |
| $\mu \mathrm{~m} \mathrm{rad}$ |  |  |  |  |  |
| Bunch length (rms) | $\sigma_{\tau}$ | 2 | ps |  |  |
| Energy spread (rms) | $\sigma_{\delta, E R L}$ | 0.02 | $\%$ |  |  |
| ERL output/input current | 11 |  |  |  | mA |

The growth of the emittance and the energy spread are shown in Fig. 6 for several values of the horizontal and vertical emittance which can be varied with the scheme of the round to flat conversion of the beam shape keeping the value of the product of horizontal and vertical emittance $[6,7]$ or the electron beam brightness. For example, if we set vertical emittance 6 pm for the 8 GeV beam, horizontal emittance is $19^{2} / 6=60 \mathrm{pm}$. With this increase of the horizontal emittance, it is expected that the beam is insensitive to degradation of horizontal emittance such as radiation excitation or CSR of which effect we neglected here. The increase of the brightness as the increase in the number of turns is calculated and is shown in Fig. 7 and 8. The calculation was performed with a standard SPring-8 undulator [8] for the 8 GeV beam and a short period hybrid undulator [9] for the $6-\mathrm{GeV}$ beam. At 8 GeV , the increase of the brightness slightly saturates because of the degradation of the beam quality. Small improvement by the flat beam is observed in Fig. 8.


Figure 6: The increase of the emittance and the energy spread by radiation excitation in the ring of the energy of 8 GeV (left) and 6 GeV (right).


Figure 7: Increase of brightness of the fundamental radiation of the undulator as the number of turns, for 8 GeV (left) and 6 GeV (right). The vertical unit for brightness is photons $/ \mathrm{sec} / \mathrm{mm}^{2} / \mathrm{mrad}^{2} / 0.1 \% \mathrm{BW}$.


Figure 8: Increase of brightness of 5-th order radiation of the undulators as the number of turns for 8 GeV (left) and 6 GeV (right).

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