

GENERATION OF OPTICAL ORBITAL ANGULAR MOMENTUM IN A FREE-ELECTRON LASER

E. Hemsing*, A. Marinelli and J. B. Rosenzweig, PBPL-UCLA, USA

Abstract

A simple scheme to generate intense light with orbital angular momentum in a free-electron laser is described. The light is generated from a helically pre-bunched beam created in an upstream modulator. The beam energy is tuned to maximize gain in the higher-order mode which reaches saturation well before the spontaneous modes driven by noise are amplified.

INTRODUCTION

Light with orbital angular momentum (OAM) has numerous current and potential uses in modern research[1]. In addition to spin angular momentum associated with the field polarization, light with a helical phase along the axis of propagation also carries an orbital component of angular momentum that can be imparted to sample, entangled, or used to rotate or spin micromechanical devices[2]. Traditionally, light with OAM has been generated in a variety of ways using elements inserted in the optical beam path. However, at shorter wavelengths and high peak power like light produced in modern x-ray FELs, such mode conversions may not be practical or feasible. Accordingly, here we investigate the possibility of generating light with OAM using a modulator/radiator arrangement that generates the OAM optical mode in situ[3]. This differs from a related scheme that relies on the harmonic emission from helical undulators[4]. The modulator section is used to helically modulate the e-beam which, after passing through a longitudinally dispersive section enters the undulator to radiate coherent OAM light. Such an arrangement might be of practical interest in general for users of x-ray FELs who require higher order optical modes. Simulations indicate x-ray OAM modes with peak power on the order of gigawatts can be obtained in this manner.

Helical Energy Modulation

In the linear regime, the evolution of the electron beam energy and phase as it interacts with helical magnetostatic modulator fields and laser fields near the harmonic resonance h is given by

$$\begin{aligned} \frac{d\eta}{dz} &= \frac{eK}{\gamma^2 mc^2} \text{Re} \left[iE^{(h)}(\mathbf{x}) e^{i\psi_0^{(h)}} \right] \\ \frac{d\psi^{(h)}}{dz} &= 2hk_w\eta + \frac{d}{dz}\varphi^{(h)}(\mathbf{x}) \end{aligned} \quad (1)$$

where $\eta = (\gamma - \gamma_0)/\gamma_0 \ll 1$ is the energy deviation of the electron from the resonant energy $\gamma_0 = \sqrt{(1 + K^2)k/2hk_w}$, $\gamma^2 = \gamma_z^2(1 + K^2)$ is the longitudinal relativistic factor, $K = eB/mck_w$ is the undulator parameter, $k = 2\pi/\lambda$ is the wavenumber of the EM field, $k \gg k_w = 2\pi/\lambda_w$ is the wavenumber of the modulator magnetic field and $d\varphi^{(h)}(\mathbf{x})/dz$ is a placeholder for additional phase contributions $\varphi^{(h)} = \text{Arg} [iE^{(h)}]$ that depend on the characteristics of the input electromagnetic field. The total harmonic phase is

$$\psi^{(h)}(\mathbf{x}) = \psi_0^{(h)} + \varphi^{(h)}(\mathbf{x}), \quad (2)$$

where the 1D ponderomotive phase is $\psi_0^{(h)} = kz + hk_w - ckt$. It has been assumed that the normalized modulator field is much larger than the normalized EM fields: $K \gg K_f = e|E^{(h)}|/mc^2k$, and that the EM fields are not modified by the electron beam.

Helical microbunching of the e-beam occurs as the initially unmodulated beam interacts with an axisymmetric laser field at harmonics in the helical modulator[3]. The harmonic interaction results from the additional oscillatory components of the electron interacting in the transversely varying input field $E(\mathbf{x})$ given by

$$E^{(h)}(\mathbf{x}) = \frac{1}{(h-1)!} \left[\frac{\pm iK}{\sqrt{2}k_w\gamma} e^{\mp i\phi} \left(\partial_r \mp \frac{i}{r} \partial_\phi \right) \right]^{h-1} E(\mathbf{x}). \quad (3)$$

This is a first order Taylor expansion of the field, for each harmonic, in the neighborhood of an electron with transverse position $\mathbf{x}_\perp = \bar{\mathbf{x}}_\perp + \text{Re}[\tilde{\mathbf{x}}_\perp e^{-ik_w z}]$. The \pm sign reflects the right- (upper sign) or left- (lower sign) handedness of the helical modulator fields. Only the oscillatory terms near resonance have been retained in the expansion. It has been assumed that the wiggle amplitude of the electron $|\mathbf{x}_w| \simeq K/k_w\gamma$ is governed primarily by the undulator field and is small. The electron only samples a small, local region of the input electric field and $\mathbf{x}_\perp \simeq \bar{\mathbf{x}}_\perp$.

For simplicity, let us assume that the input laser field is Gaussian transversely, has a pulse length much longer than the e-beam, and diffracts negligibly over the length of the modulator section. For a right-handed modulator operating at the second harmonic, the resonant ponderomotive field in Equation (3) is then given as

$$E^{(2)}(r, \phi) = \frac{r}{w_0^2} \sqrt{\frac{P\mu_0 c}{\pi w_0^2}} \left(\frac{-iK}{\sqrt{2}\gamma k_w} \right) e^{-i\phi - r^2/w_0^2} \quad (4)$$

where w_0 is the laser spot size and P is the power of the

* ehemsing@physics.ucla.edu

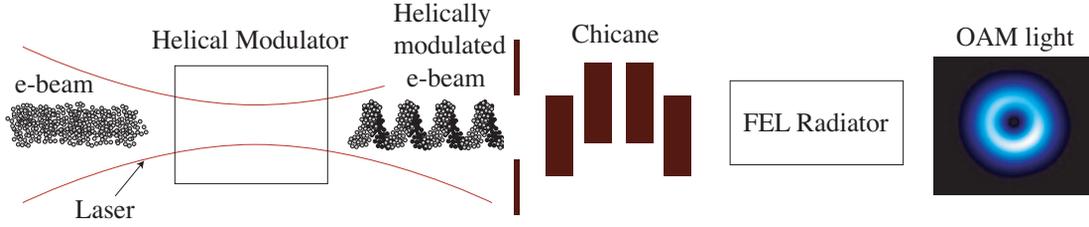


Figure 1: Arrangement for generating OAM light in an FEL. The e-beam is helically modulated via the 2nd harmonic interaction in the initial modulator stage.

input laser. The evolution equations in (1) are thus

$$\begin{aligned}\frac{d\eta}{dz} &= \frac{a(r)}{w_0} \cos(\psi - \phi) \\ \frac{d\psi}{dz} &= 4k_w\eta\end{aligned}\quad (5)$$

where $\psi = \psi^{(2)}$ and $a(r) = \frac{eK^2r}{\gamma^3mc^2k_w w_0^2} \sqrt{\frac{P\mu_0c}{2\pi}} e^{-r^2/w_0^2}$ is the energy amplitude coefficient that depends on the radial coordinate. Note that the maximum modulation is obtained at $r = w_0/\sqrt{2}$, such that only electrons off-axis are modulated in energy.

Following the small-signal gain analysis, we expand both the energy and phase to increasing orders of a small parameter: $\psi = \psi_0 + \epsilon\psi_1 + \dots$ and $\eta = \eta_0 + \epsilon\eta_1 + \dots$. To lowest order, the electron energy is a constant, with the energy deviation given by $\eta_0 = \Delta$, referred to as the detuning. To next order, the energy modulation for an initially unmodulated beam is thus given by

$$\eta(\mathbf{x}) = \Delta + \tilde{\eta} + \frac{a(r)}{4\Delta k_w w_0} \left[\sin(4\Delta k_w z + \psi_0 - \phi) - \sin(\psi_0 - \phi) \right] \quad (6)$$

where $\tilde{\eta}$ is an additional initial energy deviation and ψ_0 the initial phase position. Note that the energy modulation is a function both of the radial position r and the azimuthal coordinate ϕ . As a result, the modulation is helical along the longitudinal axis, and only the electrons off the optical axis are modulated. It is important to keep in mind that the effective energy spread (averaged over a period) induced by the modulation is $\sqrt{\langle \eta^2 \rangle} = [(\Delta + \tilde{\eta})^2 + \frac{a(r)^2}{2} \text{sinc}^2(2k_w \Delta z)]^{1/2}$, which must be much less than the FEL parameter ρ to maximize the coherent emission in the downstream radiator.

At zero beam detuning $\Delta = 0$, the initial phase can be written in terms of the co-moving beam coordinate $s = z - v_0 t$ as $\psi_0 = ks/\beta_z = k_b s$ where $k_b = 2\pi/\lambda_b$ is the microbunching wavenumber. A modulator of length L_m thus leads to a sinusoidal energy modulation of the form

$$\eta(r, \phi; s) = \tilde{\eta} + a(r) \frac{L_m}{w_0} \cos(k_b s - \phi) \quad (7)$$

where $\tilde{\eta}$ is the initial energy deviation of the electron and η is the final energy deviation at the modulator exit.

02 Synchrotron Light Sources and FELs

A06 Free Electron Lasers

Helical Density Modulation

The energy modulation can be transformed into a density modulation through a longitudinally dispersive section such as a magnetic chicane downstream of the modulator. The density bunched helical beam then acts as a seed for coherent emission of OAM light in the subsequent radiator (See Figure 1). The simple dispersive section is characterized by an R_{56} which converts the longitudinal coordinate s at the entrance of the chicane into s' , the coordinate at the exit of the chicane via

$$s' = s + R_{56}\eta \quad (8)$$

Using (7) and (8), the initial energy is related to the final energy and the final electron position in the bunch by $\tilde{\eta} = \eta - a(r) \frac{L_m}{w_0} \cos(k_b s' - k_b R_{56}\eta - \phi)$.

To calculate how the e-beam evolves from the modulator entrance to the exit of the chicane, we can define an arbitrary axisymmetric initial distribution function $f_0(r, \tilde{\eta})$. The distribution satisfies $\int f_0 r dr d\tilde{\eta} d\phi = 1$. Let us also define a helical microbunching factor of the final distribution

$$b_l = \left\langle \int f(r, \eta, \phi, \psi') e^{i\psi' - il\phi} r dr d\eta d\phi \right\rangle \quad (9)$$

which quantifies the extent to which the e-beam is density modulated into the discrete helical mode l after the dispersive section. The brackets represent averaging over the phase coordinate $\psi' = k_b s'$. The final variables can be transformed back into the initial variables using both $d\psi' d\eta = d\psi d\tilde{\eta}$ and the invariance of the distribution function, $f_0(r, \tilde{\eta}) = f(r, \eta, \phi, \psi')$. The bunching factor at the exit of the chicane is therefore determined with the integral

$$b_1 = 2\pi i \int f_0(r, \tilde{\eta}) e^{ik_b R_{56}\tilde{\eta}} J_1 \left(k_b R_{56} a(r) \frac{L_m}{w_0} \right) r dr d\tilde{\eta} \quad (10)$$

for which the only azimuthal bunching mode that is nonzero is $l = 1$, as one might expect. Even for a simple transverse distribution, this integral does not simplify in a straightforward way due to the dependence of $a(r)$ on r in the Bessel function argument. For a Gaussian beam, we consider a function of the form $f_0 = (2\pi\sigma_x^2 \sqrt{2\pi\sigma_\eta^2})^{-1} \exp(-r^2/2\sigma_x^2 - \tilde{\eta}^2/2\sigma_\eta^2)$ where σ_η is the relative rms energy spread and σ_x is the transverse rms

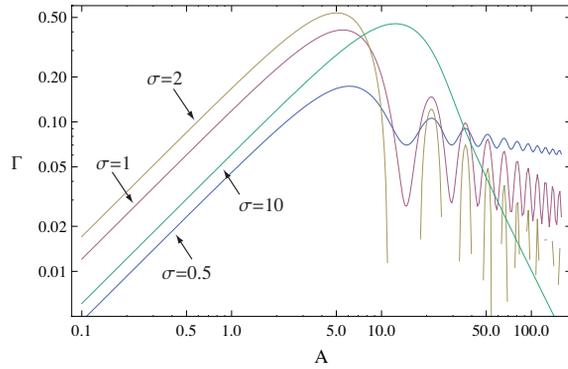


Figure 2: Transverse contribution to the helical bunching factor versus the parameter A . The ratio of the laser spot size to the e-beam is $\sigma = w_0/\sigma_x$.

beam size. The bunching factor is then

$$|b_1| = e^{-(k_b R_{56} \sigma_\eta)^2 / 2} |\Gamma| \quad (11)$$

where $\Gamma = \sigma_x^{-2} \int \exp(-r^2/2\sigma_x^2) J_1(A \frac{r}{w_0} e^{-r^2/w_0^2}) r dr$ and

$$A = \frac{eK^2 k_b R_{56} L_m}{\gamma_0^3 m c^2 k_w w_0} \sqrt{\frac{P \mu_0 c}{2\pi}} \quad (12)$$

Figure 2 shows how the transverse contribution to the bunching factor Γ varies with the parameter A . Three different ratios of the laser spot size to the e-beam size $\sigma = w_0/\sigma_x$ are shown. The maximum value of Γ is obtained for $A \simeq 5 - 12$, with larger values corresponding to larger σ . Clearly, for the proper choice of running conditions, the parameter A can be tuned to yield a reasonable value of bunching Γ . One only needs the total bunching factor b_l to be a few percent at the radiator entrance for significant coherent emission, provided the betatron wavelength is longer than a few FEL gain lengths to preserve the helical structure. One of the dominant limiting factors for short-wavelengths is the relative energy spread σ_η , which places strict limitations on the available bunching factor in Equation (11).

Coherent Emission in an FEL

Having exited the chicane with a density bunching factor on the order of a few percent, the e-beam is injected into an undulator that is tuned to emit light at the wavelength of the microbunching structure. Since the emission is approximately the same wavelength as that of the seed laser in the modulator section, the entire setup acts as a ‘mode converter’ that transforms the initially transversely gaussian laser pulse into an OAM mode. It is worth noting that, in addition to the helical bunching, intrinsic shot noise gives a non-zero bunching factor for other modes that can be amplified as SASE. Thus, one needs to make the helical bunching factor sufficiently large to insure that the OAM mode dominates during startup up to saturation. In modern devices, a few percent is typically more than

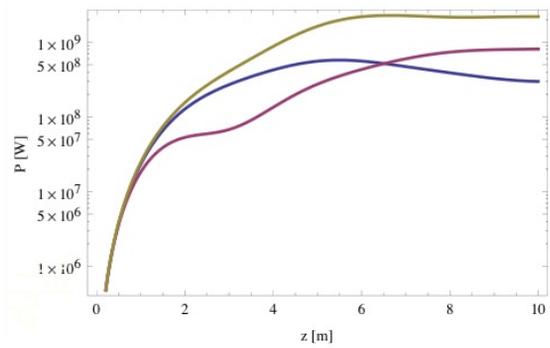


Figure 3: Coherent emission of an $l = 1$ OAM mode in an FEL, generated by helical bunching on e-beam. The curves are: $\Delta=0\%$ (blue), $\Delta=0.25\%$ (red), and $\Delta=0.5\%$ (yellow).

adequate. Figure 3 shows the OAM power emitted in an FEL predicted by time-independent numerical simulations from Genesis[6]. Parameters are; $b_1 = 8\%$, $I_0 = 1kA$, $\epsilon_{nx} = 1\mu m$, $K = 2.76$, $\lambda_w = 1.96cm$ and $\sigma_\eta = 0.05\%$. Note that this example requires initial modulation with a $\sim \lambda = 10$ nm seed which might be supplied by an upstream FEL process, perhaps even using the same e-beam (as long as the energy spread is kept low enough to allow coherent radiation in the downstream OAM undulator). Depending on the detuning of the microbunching wavelength with respect to the resonant wavelength of the OAM undulator, the emitted power can be as high as 2 GW in this scenario. In each case, the bunching factor at the fundamental mode $l = 0$ stays well below 1% through the interaction length, indicating that the fundamental mode has no growth and that the emitted optical mode is essentially a pure $l = 1$ mode.

ACKNOWLEDGMENTS

This research is supported by grants from Department of Energy Contract Nos. DOE DE-FG02-07ER46272 and DE-FG03-92ER40693 and Office of Naval Research Contract No. ONR N00014-06-1-0925.

REFERENCES

- [1] L. Allen, et al., *Phys. Rev. A*, 45(11):8185–8189, June 1992.
- [2] L. Allen, et al., *Optical angular momentum*. Institute of Physics Pub., 2003.
- [3] E. Hemsing, et al., *Phys. Rev. Lett.*, 102(17):174801, 2009.
- [4] Shigemi Sasaki and Ian McNulty. *Phys. Rev. Lett.*, 100(12):124801, 2008.
- [5] E. Hemsing, et al., *Physical Review A*, 77(6):063831, 2008.
- [6] S. Reiche. GENESIS 1.3: a fully 3D time-dependent FEL simulation code. *NIMA*, 429:243–248, June 1999.