ABCI-BASED ANALYTICAL MODEL FOR CALCULATING THE TRANSVERSE KICK FACTOR IN AXI-SYMMETRIC STEP-OUT TRANSITION

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Abstract

Step-out transition is one of the most frequent component, commonly used on the new generation light source facilities where very short and dense electron bunches are considered. The numerical calculation of the short-range wake at this type of transition requires a spatial mesh size equal to a fraction of bunch length. This calculation becomes for a very short bunch, e.g. $\sigma = 25$ μm, very time consuming due to the large number of mesh points required. On the other hand, the available analytical models that calculate the transverse wake field are applicable only on a narrow range of bunch lengths. We developed an ABCI-based analytical model that can calculate accurately the kick factor. The advantage of this model is quick, accurate and covers wide range of rms bunch lengths (up to $\sigma = 1000$ μm). The model also covers a wide range of beam pipe ratio b/a.

INTRODUCTION

Stupakov G. and Bane K.L. [1,2] studied the wakes and impedances of any non-axisymmetric transitions of any arbitrary shape using the optical approximation. As a special case of their studies, the transverse impedance of the step-out transition is given by:

$$Z_\perp(\omega) = \frac{Z_0 c}{\omega \pi a^2}\left(1 - \frac{a^2}{b^2}\right) \qquad (1)$$

Since the transverse wake function is related to the transverse impedance through inverse Fourier transform, then the transverse wake function is given by step function:

$$w_\perp(s) = \frac{Z_0 c}{2\pi a^2}\left(1 - \frac{a^2}{b^2}\right), \quad s > 0 \qquad (2)$$

The transverse wake is then simply obtained by the convolution of the charge density of Gaussian distribution with the wake function which implies that the transverse wake has a cumulative distribution shape at any bunch length and the kick factor, which is given by the integral of the wakefield weighted by the charge density distribution, is then independent of bunch length or frequency and it has a fixed value for all bunch lengths. This means, as indicated also in [2], that the optical model breaks down at long bunches. As an example, the transverse wake of step-out transition with beam pipe radii a = 5 mm and b = 13.5 mm is evaluated numerically by ABCI code [3] and analytically using eq. 2 at different Gaussian bunch lengths as shown in Fig.1.

In this paper, we studied the transverse wakes of step-out transition of an arbitrary beam pipe ratio using ABCI code to find an analytical model for calculating the kick factor based on ABCI code rather than the existing

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analytical model that is applied at very short bunches only. The main two advantages of this model is: a) accurately evaluating the kick factor on wide range of bunch lengths and beam pipe ratio, b) quick alternative tool of the time domain codes that consume too much time in calculating the kick factor due to high mesh points needed at very short bunch lengths and large step-out pipes radii.
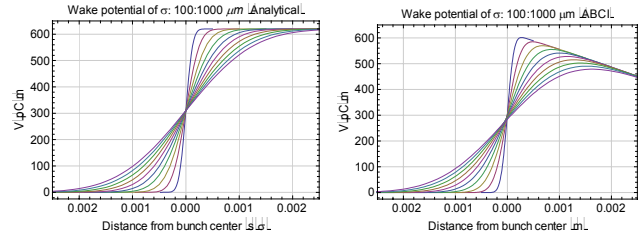

Figure1: transverse wakes of step-out transition as a function of bunch length using analytical optical approximation (left) and ABCI code (right).

MODEL APPAROACH

We run ABCI at a different input pipe radii starting from *a = 2.5 mm* and up to *10 mm*. For each value of the smallest radius *a* we changed the other beam pipe radius *b*. For each value of *b* we plot the kick factor as a function of the rms bunch length σ. In this context, the smallest value of *b* was *4 mm* at *a= 2.5 mm* and the greatest value was *72 mm* at *a =10 mm*. For all simulations, we noticed that the kick factor has always a linear relationship with respect the rms bunch length (σ) whatever the beam pipe ratio *(a/b)* is. Fig.2 represent an example of tremendous number of simulation at fixed beam pipe radius *a = 3.5 mm*. The figure shows very well the aforementioned linear relationship and the linear fit of different kick factors at different beam pipe radius *b*.
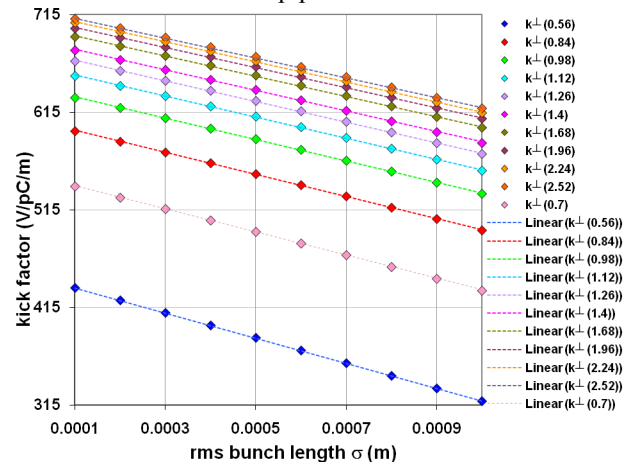

Figure 2: kick factor of step-out transition at input radius *a = 3.5 mm* as a function of rms bunch length and different output pipe radius *b* shown in brackets.

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D05 Instabilities - Processes, Impedances, Countermeasures

It is worthwhile to mention that the linear relationship of the kick factor as a function of the bunch length is depending mainly on the mesh resolution where the mesh size should be small enough to avoid numerical dispersion especially at very short bunches. Coarse mesh size will make kick factor at very short bunches almost independent of bunch length. On the other hand the fine mesh size is mandatory since the excited frequency spectrum depends strongly on the mesh size. For that reason we take the mesh size to be $\sigma/10$ and $\sigma/20$ in vertical and longitudinal directions respectively.

The kick factors shown in Fig. 2 are fitted to the linear formula:

$$K\perp = A_0 (a,b) \ \sigma + B_0(a,b) \qquad (3)$$

Where the fitting parameters $A_0(a,b)$, $B_0(a,b)$ are function of step-out transition radii a,b. Table 1 represents their values at different beam pipe radius b while Fig. 3 shows their plot. These fitting parameters are then fitted to the formulae (4) and (5) and the fitted values are shown by dashed lines in Fig. 3:

$$A_0(a,b) = A_1(a) \times e^{-\frac{b}{A_2(a)}} + A_3(a) \times e^{-\frac{b}{A_4(a)}} + A_5(a) \qquad (4)$$

$$B_0(a,b) = B_1(a) \times e^{-\frac{b}{B_2(a)}} + B_3(a) \times e^{-\frac{b}{B_4(a)}} + B_5(a) \qquad (5)$$

Table 1: Fitting parameters $A_0(a,b)$, $B_0(a,b)$

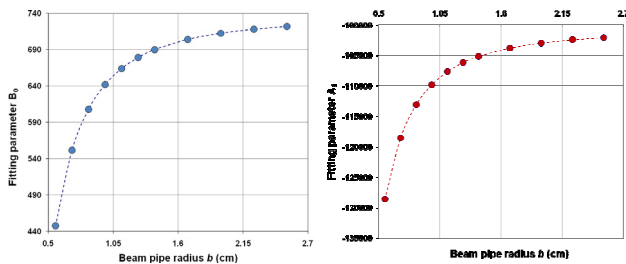| b (cm)	$A_0(a,b)$	$B_0(a,b)$
0.56	-128576	447.77
0.7	-118558	551.49
0.84	-113091	607.84
0.98	-109818	641.82
1.12	-107661	663.85
1.26	-106176	678.97
1.4	-105152	689.77
1.68	-103818	703.9
1.96	-102976	712.37
2.24	-102430	717.87
2.52	-102097	721.67

Figure 3: Fitting parameters $A_0(a,b)$, $B_0(a,b)$ (circles) and their fittings (dashed lines) as a function of output radius b when the input radius $a = 3.5$ mm.

As noticed, each of the fitting parameters $A_0(a,b)$, $B_0(a,b)$ have their own new 5 fittings parameters which are function in the pipe radius a only. The same procedure has been repeated at different values of pipe radius a to obtain the fitting parameters $A_0(a,b)$, $B_0(a,b)$ at each a then find their subsequent fitting parameters $A_1(a)-A_5(a)$ and $B_1(a)-B_5(a)$. Table 2 represents such fitting parameters at different values of a while Fig. 4 shows the fitting parameters $A_1(a)-A_5(a)$ and $B_1(a)-B_5(a)$ with their fittings as a function of pipe radius a. Equations from (6.1) to (6.10) represent the final mathematical formulation of the fitting parameters $A_1(a)-A_5(a)$ and $B_1(a)-B_5(a)$.

Table 2: Fitting parameters $A_1(a)$: $A_5(a)$ and $B_1(a)$:$B_5(a)$ at different values of pipe radius a.

a (mm)	$A_1(a)$	$A_2(a)$	$A_3(a)$	$A_4(a)$	$A_5(a)$
2.5	-577100	0.15	-57890	0.5661	-283400
3.5	-306032	0.1769	-35726.7	0.60176	-101571
5	-153195	0.21326	-19742.1	0.72849	-33630.1
7.5	-54214.1	0.29982	-7026.01	1.03489	-9745
10	-13517.8	0.57124	-1183.31	2.18164	-4042.89

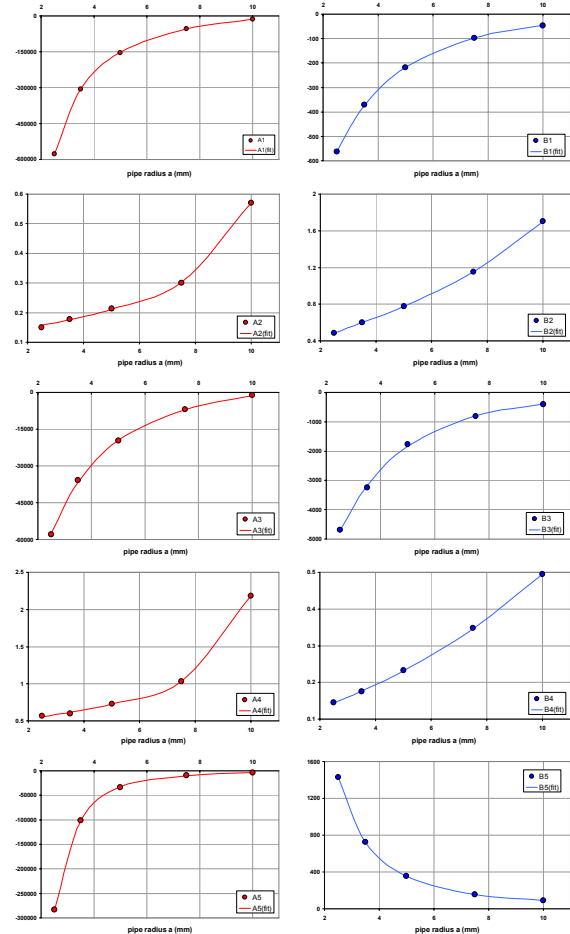| a (mm)	$B_1(a)$	$B_2(a)$	$B_3(a)$	$B_4(a)$	$B_5(a)$
2.5	-561.92	0.48491	-4686.7	0.1447	1428.9
3.5	-371.44	0.60242	-3245.3	0.17513	727.05
5	-218.38	0.7783	-1770.2	0.23296	354.48
7.5	-98	1.15	-815.7	0.348	157.4
10	-46.432	1.702	-403.68	0.49456	88.817

Figure 4: fitting parameters $A_1(a)$:$A_5(a)$ (red circles) and $B_1(a)$: $B_5(a)$ (blue circles) with their fittings (solid lines) as a function of pipe radius a.

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$$A_1(a) = -7.386 \times 10^6 \, e^{-1.409 \times a} - 8.628 \times 10^5 \, e^{-0.3321 \times a} + 1.792 \times 10^4 \tag{6.1}$$

$$A_2(a) = 0.118 \, e^{0.114 \times a} + 4.244 \times 10^{-5} \, e^{0.8469 \times a} \tag{6.2}$$

$$A_3(a) = 5.48 \times 10^5 \, e^{-0.002425 \times a} - 1.767 \times 10^5 \, e^{-0.5309 \times a} - 5.618 \times 10^5 \tag{6.3}$$

$$A_4(a) = 0.4281 \, e^{0.102 \times a} + 1.784 \times 10^{-4} \, e^{0.8626 \times a} \tag{6.4}$$

$$A_5(a) = -6.246 \times 10^6 \, e^{-1.342 \times a} - 1.661 \times 10^5 \, e^{-0.3747 \times a} \tag{6.5}$$

$$B_1(a) = -1.394 \times 10^3 \, e^{-0.9765 \times a} - 935.8 \times 10^6 \, e^{-0.3013 \times a} \tag{6.6}$$

$$B_2(a) = 0.3589 \, e^{0.1556 \times a} - 0.4045 \, e^{-0.88884 \times a} \tag{6.7}$$

$$B_3(a) = -1.054 \; 10^4 \, e^{-0.409 \times a} - 1.986 \times 10^3 \, e^{-0.4099 \times a} - 203.6 \tag{6.8}$$

$$B_4(a) = 1.566 \, e^{0.07243 \times a} - 1.488 \, e^{0.06093 \times a} \tag{6.9}$$

$$B_5(a) = 9002 \, a^{-2.009} \tag{6.10}$$

MODEL VALIDITY

Now we have all parameters needed to calculate the kick factor, for example for any value of pipe radius a the fitting parameters $A_1(a)$-$A_5(a)$ and $B_1(a)$-$B_5(a)$ are evaluated using equations from (6.1) to (6.10) then introducing pipe radius b in equations 4 and 5 we get the fittings parameters $A_0(a,b)$, $B_0(a,b)$ which can be used in turn in Eq. 3 to find the kick factor. To check the model validity we run ABCI code at different bunch lengths and pipe ratios other than those already used in model formulation. Fig. 5 shows ABCI kick factors of different 4 step-out transitions in comparison with those obtained by the model we just derived. Apparently, the model is in excellent agreement with ABCI data.
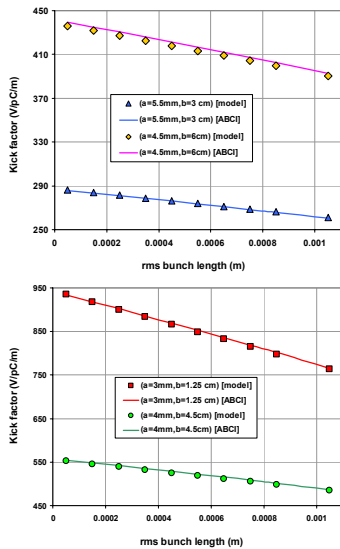

Figure 5: Kick factor of four step-out transitions with different pipe radii ratio. Scattered data are.

WORK IN PROGRESS

Since the step-out transition is a special case of taper-out transition with tapering angle $\theta = 90^0$, then we decided to use the same approach described in this paper to extend the model to cover the taper-out transition of any arbitrary tapering angle of the wall connects the two beam pipes. The model will also include the wake potential plotting.

CONCLUSION

Kick factor of long bunches passing a step-out transition can not be calculated accurately using the current available analytical model since it is applicable only at very short bunches where the optical approximation is applied. Even at short bunches the available codes take too much time in calculation process. Using ABCI code we derived an analytical model that overcame the aforementioned two shortcomings. The model calculates accurately and very quickly the kick factor of any step-out transition of arbitrary pipes radii ratio and it is operating perfectly on a wide range of bunch length starting from point charge (i.e. $\sigma = 0$ μm) and up to $\sigma = 1000$ μm. The reader that wants to check the validity of this model should consider the mesh size mentioned in the second section. We believe that this model in its final form will save too much time wasted in transverse wake calculation of one of the most frequently used components in light source facilities i.e. step/taper out transitions.

ACKNOWLEDGEMENT

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[1] Bane, K.L., and G. Stupakov "Impedance calculations of non-axisymmetric transitions using the optical approximation". Phys. Rev. ST Accel. Beams 10, 074401, 2007.

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[3] ABCI website: http://abci.kek.jp/abci.htm.