# SHORT RANGE WAKEFIELDS STUDIES OF STEP-OUT AND TAPER-OUT TRANSITIONS ADJACENT TO X-BAND LINAC IN FERMI@ELETTRA

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## *Abstract*

FERMI@ELETTRA is a single pass FEL Facility in construction at the ELETTRA Laboratory in Trieste. To linearize the beam longitudinal phase space, it is planned to use a short X-band accelerating structure installed before the first bunch compressor. Since both the end tubes of the structure have a reduced radius of 5.0 mm, much smaller than the 13.5 mm radius of the beam pipes before and after the structure, a transition, either stepped or tapered, will be necessary between the two components. Using the ABCI code, we have investigated the short range wake fields at the step-out and taper-out transitions and we have compared them with some conventional analytical models. We have developed specific ABCI-based analytical models that simulate accurately the short range wake field for a wide range of rms bunch lengths (σ: 100 - 1000 μm).


## LONGITUDINAL WAKEFIELDS

Fig. 1 represents the two possible transitions understudy. Due to limited space, the distance allowed for tapered transition is only 2.882 cm.

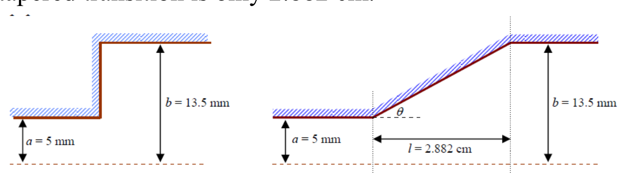


Figure 1: Lay out of the step-out and tapered-out transitions of X-band structure to the beam tubes.

### *Step-out transition*

For very short bunches ($\sigma_z << a$) it has been found that the wake potential is resistive [1-3], i.e. the shape of the wake potential is the same as the bunch distribution [4]. This means that the real part of the impedance is constant at high frequency and the imaginary part is zero. For a Gaussian bunch the wake is given by:

$$\overset{step}{W_l}(s) = \frac{Z_0 c \ln(b/a)}{\sqrt{2}\pi^{3/2}\sigma_z} e^{-\frac{s^2}{2\sigma_z^2}} \quad (1)$$

Where $s$ is the position within the bunch and $Z_0 = 377\Omega$ and $c$ the speed of light; the real and imaginary parts of the impedance at high frequency are given by:

$$\mathrm{Re}\, Z_l = \frac{Z_0}{\pi} \ln\left(\frac{b}{a}\right), \; Im\, Z_l \approx 0 \quad (2)$$

The longitudinal loss factor in terms of $Z_l$ is given by:

$$\overset{step}{K}_{loss} = \frac{\mathrm{Re}\, Z_l c}{2\sqrt{\pi}\sigma_z} \quad (3)$$

ABCI code [5] has been used to check the compliance of our step transition with equations (1-3). Fig. 2 shows both the real and imaginary impedances at two distinguished bunch lengths. As noticed, the impedances have a resonance just after beam pipe cut-off and reach asymptotic value ($\mathrm{Re}Z_l \approx 116\ \Omega$, $\mathrm{Im}Z_l \approx 0\ \Omega$) at high frequency. Eq. 3 slightly overestimates the real impedance (119.2 Ω) which in turn overestimates the loss factor. Moreover, and as concluded from Fig.2, the analytical model is perfectly valid for very short bunches ($\sigma_z << a$), where the diffraction model [6,7] is applied. At long bunches ($\sigma_z \leq a$) the impedance is not purely resistive and it has a reactive (inductive) component which means that the tail of the long bunch will gain energy by the wake field. This is clearly shown in Fig. 3 for the same two bunches. Accordingly, the same analytical model (Eq. 1) could be used to accurately evaluate the wakes on a wide range of bunch lengths if it is modified to compensate: a) the slight overestimation of peak potentials at short bunches, b) the inductive effect at long bunches.

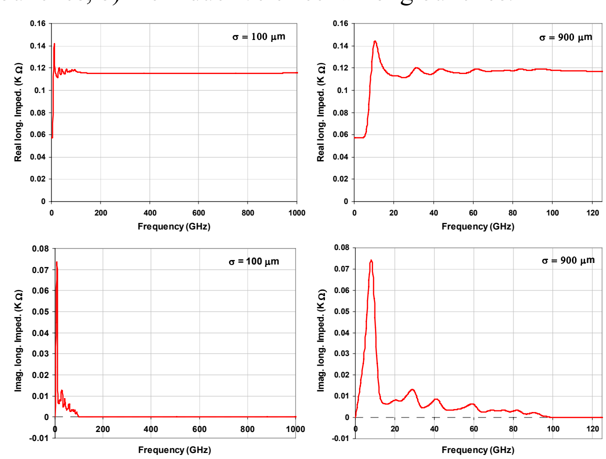


Figure 2: Real and imaginary impedances of the step-out transition at 100 μm and 900 μm rms bunch lengths

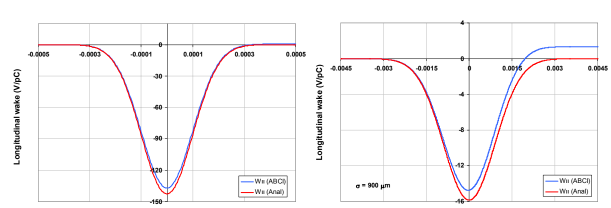


Figure 3: Numerical and analytical longitudinal wake potentials of the step-out transition at 100 μm and 900 μm rms bunch lengths

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This could be done by investigating the deviation of the analytical model from the ABCI data. Fig. 4 shows the error evolution in the longitudinal wakes as a function of the bunch length. It is quite clear that at very short bunches the error is mainly localized around the beam center; as the bunch length increases the error is extended to the bunch tail till eventually approaches the sigmoid function attitude. Accordingly, a sigmoid function will be added to the analytical model (Eq. 1) to compensate the aforementioned error. The new modified analytical model has the form:

$$\overset{step}{W_l}(s)_m = A(\sigma_z)\times W_l(s)+\frac{B(\sigma_z)}{1+e^{(C*(s-D))}} \qquad (4)$$

ABCI data has been fitted using the new model and the fitting parameters have been evaluated for each bunch length. Parameters *C = -4366.28* and *D = 0.000589* are fixed for all bunch lengths while $A(\sigma_z)$ and $B(\sigma_z)$ are shown in Fig. 5 and given by equations (5,6):

$$A(\sigma_z)=0.96865-0.00599\ e^{(0.00183\ \sigma_z)} \qquad (5)$$

$$B(\sigma_z)=1.21300+1.07300\ e^{(-0.002537\ \sigma_z)} \qquad (6)$$

Fig. 6 represents the wakes calculated by the new ABCI-based model in comparison with ABCI data for different bunch lengths (σ : 100 µm – 1000 µm).

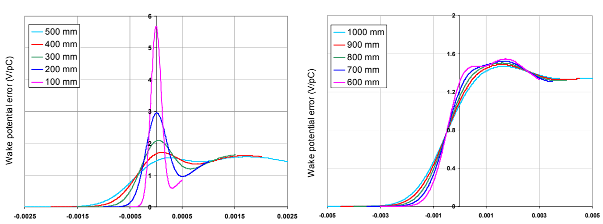


Figure 4: Deviation of Eq. 1 from ABCI data as the bunch length increases.

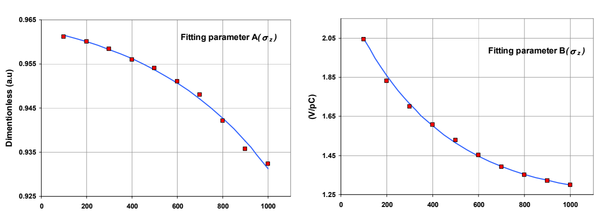


Figure 5: Fitting parameters $A(\sigma_z)$ and $B(\sigma_z)$ as a function of bunch length.

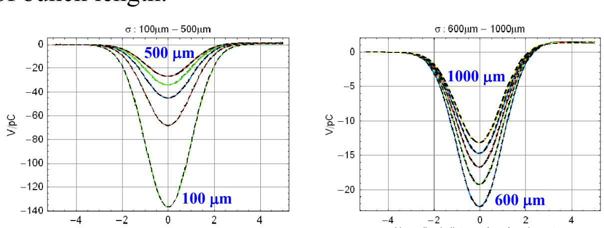


Figure 6: Longitudinal wakes calculated by the new ABCI model (black dashed) vs. ABCI (coloured) for step case.

As shown, the new ABCI-based analytical model is accurately fit to the wake of the ABCI numerical data along the entire bunch. To derive an accurate model of the loss factor we used the following equation:

$$K_{\mathrm{Loss}} = \int_{-5\sigma}^{5\sigma}\lambda(s)\ \overset{step}{W_l}(s)_m\, ds \qquad (7)$$

Where $\lambda(\sigma)$ is the bunch distribution. Comparing the loss factor obtained by Eq. 7 with that one given by Eq. 3, the new ABCI-based loss factor formula is obtained as:

$$\overset{step}{K}_{loss} = \frac{\mathrm{Re}\, Z_l c}{2\sqrt{\pi}\sigma_z}\times\left(1-0.03632\ e^{(s\ 10^{-3})}\right) \qquad (8)$$

Fig. 7 shows the loss factor obtained by ABCI data in comparison with the original model (Eq. 3) and the new ABCI-based model (Eq. 8). As shown, the new model evaluates accurately the loss factor on a wide range of bunch lengths (σ : 100 µm – 1000 µm).

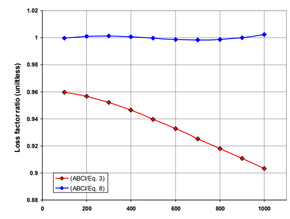


Figure7: Comparison of loss factor ABCI with respect to those calculated using Eq. 3 and Eq. 8.

### *Taper-out transition*

The impedances and wakes of slowly tapered structures have been investigated and suggested by many authors [7-9] to reduce the loss caused by step transition. As mentioned in Fig. 1, the distance allowed for tapering is only 2.882 cm making $17^0$ degree tapering angle. Since the tapering angle will not be significant unless the angle is: $\tan\theta \approx \sigma_z/a$ [4], hence the existing taper could be considered as step-out transition specially at very short bunches and as intermediate case between step-out and taper out transitions at long bunches. Accordingly, Eq. 1 will be used as a start point for evaluating the wakes and Eq. 9 [7, 10] for the loss factor calculation:

$$\overset{tap}{K}_{loss} = \frac{\mathrm{Re}\, Z_l c}{2\sqrt{\pi}\sigma_z}\left(1-\frac{\tilde{\eta}}{2}\right) \qquad (9)$$

$$\tilde{\eta} = \min\left\{1, \frac{l\sigma}{(b-a)^2}\right\} \qquad (10)$$

We followed the same approach of step-out transition case; based on the deviation of Eq. 3 from ABCI data, the new ABCI-based model of the wake potential in the taper-out transition case is suggested as follow:

$$\overset{tap}{W_l}(s)_m = E(\sigma_z)\times W_l(s)+F(\sigma_z)\ e^{\left(-\frac{(s-g)^2}{2h^2}\right)} \qquad (11)$$

Parameters *g = -0.0025* and *h= 0.0015* are fixed for all bunch lengths while $E(\sigma_z)$ and $F(\sigma_z)$ are shown in Fig. 8 and given by equations (12,13):

$$E(\sigma_z) = 0.8912 + 0.1069\ e^{-\left(\frac{\sigma_z - 551}{656}\right)^2} \qquad (12)$$

$$F(\sigma_z) = 10.1\ e^{-(0.000645\sigma_z)} - 10.9\ e^{-(0.00415\sigma_z)} \qquad (13)$$

Fig. 9 shows the wakes calculated by the new ABCI-based model for taper case in comparison with ABCI data for different bunch lengths (σ : 100 μm – 1000 μm). Apparently, the new ABCI-based analytical model fits accurately to the wake of the ABCI numerical data along the entire bunch. Worthy to note that in this case the loss factor given by equations [9, 10] is reliable within ±2%.

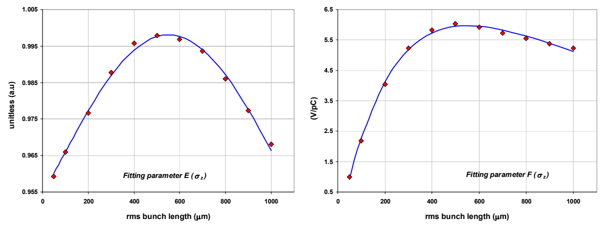


Figure 8: Fitting parameters $E(\sigma_z)$ and $F(\sigma_z)$ as a function of bunch length.

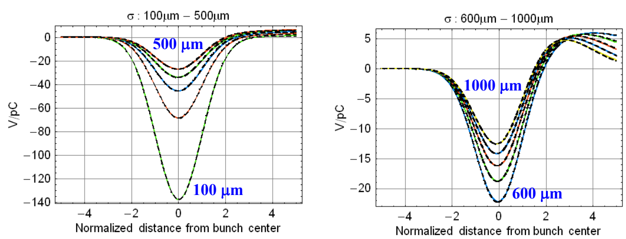


Figure 9: Longitudinal wakes calculated by the new ABCI model (black dashed) vs. ABCI (coloured) for taper case.

## TRANSVERSE WAKEFIELDS

One of the most useful uses of the Panofsky-Wenzel theorem is that by integrating the longitudinal wake function we can obtain the transverse wake function and vice versa. Accordingly, since the wake potential of bunch of particles with Gaussian distribution in step-out transition has a Gaussian pattern, then the transverse wake potential will be the integration of such Gaussian shape leading to a transverse wake potential with the shape of cumulative distribution function. Bane K.L. and Stupakov G. [11,12] studied in deep details the wakes and impedances for any non-axisymmetric transitions of any arbitrary shape using the optical approximation regime. As a special case of their studies, the transverse impedance of the step-out transition is given by:

$$Z_\perp(\omega) = \frac{Z_0 c}{\omega \pi a^2}\left(1 - \frac{a^2}{b^2}\right) \qquad 4)$$

Since the transverse wake function is related to the transverse impedance through inverse Fourier transform, hence the transverse wake function is given by:

$$w_\perp(s) = \frac{Z_0 c}{2\pi a^2}\left(1 - \frac{a^2}{b^2}\right), \quad s > 0 \qquad (15)$$

Starting with these two equations we followed the same approach of longitudinal case and we derived an ABCI based analytical model for evaluating the transverse wake fields of step/taper out transitions on wide range of bunch lengths (σ: 100μm–1000μm). Fig.10 shows the transverse wakes calculated by the new ABCI-based models in comparison to ABCI data for both out transitions.

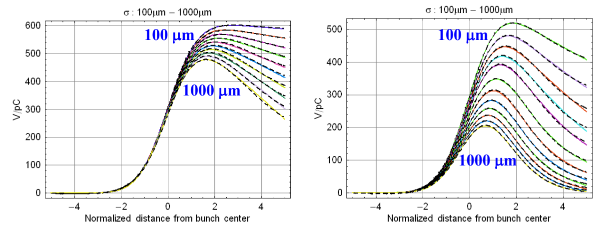


Figure10: transverse wakes calculated by the new ABCI model (black dashed) vs. ABCI (coloured) for step-out transition case (left) and taper-out transition (right).

## CONCLUSION

There is no preference to use specific transition at very short bunches. At long bunches the taper out transitions is preferred from the transverse wakes point of view only. More details will be available in PRST version.

## ACKNOLEDGEMENT

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