# ELECTRON CLOUD IN THE REGION OF WEAK VERTICAL FIELD OF THE WIGGLER

Levi Schächter, Department of Electrical Engineering, Technion, Haifa 32000, ISRAEL.

#### Abstract

In the region of weak vertical magnetic field, the longitudinal magnetic field grows rapidly and as a result, electrons generated on axis due to ionization of the remnant gas, may become trapped. Ignoring space-charge effects and the ion's effect, simulations indicate that electrons may reach the walls of the beam-chamber by gyrating along magnetic field lines during a period of time comparable with the bunch spacing (nano-seconds). In the course of their motion, they generate radiation. Spectrum's width is directly related to the cloud temperature and since collective effects may be ignored, both the total energy and the power are proportional to the number of electrons in the cloud. The spectrum is also indicative of the expansion of the cloud off-axis: the further electrons expend out of the weak magnetic field region, they encounter stronger magnetic fields and gyrate faster. Faster oscillation manifests as broadening of the emitted spectrum towards higher frequencies. In the second part we consider the interaction of a train of (M) macro-bunches with a periodic distribution of electroncloud layers each one of them characterized by a scalar dielectric function (resonance and quality factor). The energy transferred by the bunches is considered. Coherent process (radiation proportional to  $M^2$ ) may occur and the conditions for this to happen are discussed.

#### **INTRODUCTION**

As multi-GeV bunches of electrons or positrons circulate in damping rings, they generate synchrotron radiation which is essential for the particles' cooling process but when hitting the metallic walls, this radiation generates photo-electrons. An additional process, that may generate free electrons in the beam-chamber, is the ionization of the residual gas either by the particles or the radiation. A third mechanism responsible to the presence of free-electrons in the beam-chamber is secondary emission occurring when electrons impinge upon the metallic wall. Finally, stray particles from the bunch may also generate free-electrons. Known as "electron-cloud" (EC) all these free-electrons, energized directly by the bunches or by the wake-field, may have a detrimental effect on the operation of a damping ring. In fact, since in zero order, all these processes are proportional to the number of particles in each bunch, the EC effect becomes more severe as this number increases [1].

In the framework of the present report, we focus on the effect of electrons resulting from *ionization of the remnant gas in the wiggler* and specifically in the vicinity of the zero vertical magnetic field. Gyrating along the magnetic field lines, these electrons which are born in the vicinity of the chamber's axis, are trapped for extended period of time. Their motion is determined by the wiggler,

the positive ion column on axis and the train of relativistic bunches. In the first part of this report we examine the implications of the characteristic motion of the electrons subject to the first two. In the second part we harness the knowledge gained in the first part, in order to establish the effect of an *array the electron cloud layers on a train of bunches* circulating in a damping ring.

### PART I: DYNAMICS OF ELECTRONS IN SINGLE E-CLOUD LAYER

For simplicity sake, the beam-chamber is assumed to be of rectangular cross-section and the vertical and the longitudinal components of the wigglers' magnetic field components are given by

$$B_{y} = B_{0} \sum_{n=1}^{\infty} f_{n} \sin\left(2\pi n \frac{z}{L}\right) \cosh\left[\frac{2\pi n}{L}\left(y - \frac{a_{y}}{2}\right)\right],$$

$$B_{z} = B_{0} \sum_{n=1}^{\infty} f_{n} \cos\left(2\pi n \frac{z}{L}\right) \sinh\left[\frac{2\pi n}{L}\left(y - \frac{a_{y}}{2}\right)\right].$$
(1)

For simplicity sake, we consider the impact of the *first* harmonic tracing 200 macro-particle initially located at |y/L| < 0.003, |z/L| < 0.01 their initial energy being 340[eV]; the periodicity of the wiggler is L = 40[cm], the height of the beam-chamber is  $a_y = 5[\text{cm}]$  and its width is assumed to be  $a_x = 9[\text{cm}]$ . At the particular energy imparted to the ensemble, the first macro-bunch reaches the wall at  $\tau \square 1000$  corresponding to 3[nsec]. It should be pointed out that after a similar time duration, if the electrons' initial energy was 50eV, the leading macroparticle has crossed less than half the way to the wall ( $y_{\text{max}} / L = 0.062$ ). The two frames in Figure 1 reveal the essence of the result. The red dots in the center illustrate in both cases the initial distribution of particles.

The signature of the spectrum emitted by these electrons is expected to be significantly different than that emitted by photo-electrons in bends or peak vertical field of a wiggler. Consequently, this spectrum may provide us with a way to *differentiate* between the two processes: photo and secondary electrons on the one hand and electrons resulting from ionization of the remnant gas on the other hand. In principle, each eigen-mode is excited and in Figure 2 we show the contribution to the normalized spectrum of first *two modes* clearly indicating that the power peaks at the cut-off and decays exponentially at high frequencies.

In order to investigate the radiation generated by the particles, without the effect of the vacuum chamber's walls, we examine next the radiation emitted to *free space*. Figure 3 illustrates the relative effect of all three components of motion and the total radiation emitted.

#### **05 Beam Dynamics and Electromagnetic Fields**



Figure 1: Electron cloud distribution at two instants  $\tau = \Omega t = 500,1000; \ \Omega = eB_0 / m$ .

The frequency is normalized to the cyclotron frequency with  $[B_0 = 1.9T, \Omega/2\pi \Box 54GHz]$ . For a sufficiently energetic cloud (330eV), see Figure 3, electrons expand reaching high magnetic field thus gyrating and emitting radiation up to about 20GHz. Peaks occur in regions were electrons are trapped for extended periods of time. Clearly, the spectrum resulting from the horizontal motion is dominant, the spectrum due to longitudinal motion is virtually identical at low frequencies with the horizontal contribution. The latter and the vertical contributions are identical for 80% of the upper part of the spectrum. A similar trend is revealed in Figure 4, for a slightly lower average energy (250eV).

Figure 5 shows that by reducing the peak magnetic field by a factor of 1/3, the width of the spectrum shrinks accordingly and so is the total energy radiated. The peak in the spectrum is attributed to the fact that many electrons are trapped in a small region and they all contribute the same frequency.



Figure 2: The first two modes excited by the e-cloud electrons in the vicinity of one the nodes of the wiggler.



Figure 3: Normalized spectrum emitted into free-space by the ensemble of electrons consisting the electron cloud; the average initial energy is 330eV. The contribution of each component of motion is emphasized.



Figure 4: Normalized spectrum emitted into free-space by the ensemble of electrons consisting the electron cloud; the average initial energy is 250eV. The contribution of each component of motion is emphasized.

At low temperatures (10eV) electrons do not have enough energy to expand significantly, therefore they gyrate in the region of weak magnetic field and correspondingly, the spectrum they emit is only in the low frequency range as illustrated in Figure 6. Moreover, the

# 05 Beam Dynamics and Electromagnetic Fields

D05 Instabilities - Processes, Impedances, Countermeasures

total energy emitted is reduced by a factor which is proportional to the average initial energy of the electrons in the cloud. Note that for making the spectrum visible on the scale of the 330eV spectrum, we had to multiply the spectrum of the 10eV electrons by a factor of 10.



Figure 5: Normalized spectrum emitted into free-space by the ensemble of electrons consisting the electron cloud at two different peak values of the magnetic field; the average initial energy is 330eV.



Figure 6: Normalized spectrum emitted into free-space by the ensemble of electrons consisting the electron cloud for two different temperatures; the peak magnetic field is 1.9T.

## PART II: TRAIN OF BUNCHES IN AN ARRAY OF ECLOUD LAYERS

In the present section we assume that in a layer of thickness  $\Delta_{ee}$ , in the vicinity of zero vertical magnetic field, there is an e-cloud of density  $n_{ee}$ . Assuming a base pressure of  $10^{-9}$  Torr the average density of Hydrogen atoms in the vacuum chamber is  $n_{ee} < 10^{13} \text{ m}^{-3}$ . With this electron density we may define the corresponding plasma density  $\omega_{p,ee}^2 = e^2 n_{ee} / m\varepsilon_0$  which according to our previous assessment is in any case much smaller than 30MHz. The oscillatory character of the e-cloud motion is represented by a resonance denoted by  $\omega_0$  whose bandwidth is specified by a quality factor Q representing the intra-

electrons collisions. Based on these definitions, the dielectric coefficient that describes the e-cloud is

$$\varepsilon_{\rm ec}\left(\omega\right) = 1 + \frac{\omega_{\rm p,ec}^2}{\omega_0^2 + 2j\omega\omega_0/Q - \omega^2} \,. \tag{2}$$

It should be pointed out that this homogeneous and isotropic description is an *idealization* of the complex electrons' dynamics, discussed in the previous section. Its goal is to assess the zero-order effect of the cloud on a train of bunches. A train of *M* bunches each one carrying a charge  $q = eN_{el}$  and being separated by each other by  $T_{b}$ , moving at a constant velocity  $V \square c$ .

The energy transferred by the train of bunches is proportional to

$$W \propto M - 2\operatorname{Re}\left[\frac{M + 1/M}{1 - \exp(-\chi)} - \frac{1 - \exp[-(M+1)\chi]}{\left[1 - \exp(-\chi)\right]^2}\right], \quad (3)$$

wherein  $\chi = \omega_0 T_b / Q + jT_b \sqrt{\omega_0^2 + \omega_{p,ec}^2}$ . Figure 7 shows that *coherent* energy transfer  $(W \propto M^2)$  occurs at low resonant frequency  $(\omega_0 T_b \Box 1)$  and single-particle process  $(W \propto M)$  occur at high resonant frequency,  $(\omega_0 T_b \Box 1)$ . Between these two extremes, there are resonant peaks at  $\omega_0 T_b \Box 2\pi n$ .



Figure 7: Normalized energy transferred as a function of  $\omega_0 T_{\rm b}$  with the quality factor as a parameter in the left frame.

#### REFERENCES

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