SORTING CHROMATIC SEXTUPOLES FOR SECOND ORDER CHROMATICITY CORRECTION IN THE RHIC*

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Abstract

In this article, based on the contributions of the chromatic sextupole families to the half-integer resonance driving terms, we discuss how to sort the chromatic sextupoles in the arcs of the Relativistic Heavy Ion Collider (RHIC) to easily and effectively correct the second order chromaticities. We proposed a method with 4 knobs corresponding to 4 pairs of chromatic sextupole families to correct the second order chromaticities. Numerical simulation shows that this method reduces the unbalance in the correction strengths among the sextupole families and avoids the reversal of sextupole polarities and therefore yields larger dynamic apertures for the proposed RHIC 2009 100GeV pp run.

INTRODUCTION

There are 144 chromatic sextupoles in the 6 arcs in each ring of the Relativistic Heavy Ion Collider (RHIC). Before 2007, there were 12 power supplies for them in each ring. In each arc there are two independent power supplies, one for all focusing sextupoles (SFs) and one for all defocusing sextupoles (SDs). In the RHIC operation before 2007, a 2-family chromaticity correction scheme was used. The 2-family correction scheme can only correct the first order chromaticities.

During the Summer shutdown of RHIC 2006, the number of the power supplies for the arc chromatic sextupoles were doubled from 12 to 24 to allow the correction of the second order chromaticities. After doubling the power supplies, in each arc there are two power supplies for the SF sextupoles and two for the SD sextupoles. The SF and SD sextupoles in each arc are split into two SF and two SD sub-families. The chromatic sextupoles in the 3 outer arcs are sorted as 6*(SFPO, SDMO, SFMO, SDMO), while the chromatic sextupoles in the 3 inner arcs are sorted as 6*(SFPI, SDMI, SFMI, SDMI) [1]. Currently there are totally 8 sextupole families, SFPO, SDMO, SFMO, SDMO, SFPI, SDMI, SFMI, SDMI in the RHIC control system.

In the following, we first review that both first order off-momentum β-beats and second order chromaticities are related to the half-integer resonance driving terms (RDTs). Then we calculate the contributions from sextupole families to the half-integer RDTs and propose a method with 4-knobs corresponding to 4-pairs of sextupole families to correct the second order chromaticities. Numerical simulations show that the 4-knob method does reduce the unbalance in the correction strengths among the sextupole families, and avoids the reversal of sextupole polarities.

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PERTURBATION THEORY

The first order chromaticities are

$$\xi^{(1)}_{x,y} = \frac{\partial Q_{x,y}}{\partial \delta} = \frac{1}{4\pi}\oint \beta_{x,y}(s)[\mp K_1(s) \pm K_2(s) D_x(s)]ds, \quad (1)$$

where $\beta_{x,y}(s)$ are the unperturbed β functions, $D_x(s)$ is the horizontal dispersion, and $K_{1,2}$ are the quadrupole and sextupole strengths.

The first order off-momentum β-beats are

$$\frac{1}{\beta_{x,y}(s)}\frac{\partial \beta_{x,y}(s)}{\partial \delta} = \pm\frac{1}{2\sin(2\pi Q_{x,y})}\oint \beta_{x,y}(s')[\mp K_1(s') \pm K_2(s') D_x(s')] \cos(2|\phi_{x,y}(s)-\phi_{x,y}(s')| - 2\pi Q_{x,y}) ds'. \quad (2)$$

The second order chromaticities can be calculated following [2]

$$\xi^{(2)}_{x,y} = \frac{1}{2}\frac{\partial^2 Q_{x,y}}{\partial \delta^2}|_{\delta=0} = \frac{1}{2}\frac{\partial \xi^{(1)}_{x,y}}{\partial \delta}|_{\delta=0}. \quad (3)$$

Plugging Eq. (1) into Eq. (3) and considering the changes of magnetic strengths, β functions and dispersion due to the relative momentum deviation δ, we obtain

$$\xi^{(2)}_{x,y} = -\frac{1}{2}\xi^{(1)}_{x,y} + \frac{1}{4\pi}\oint[\mp K_1 \pm K_2 D_x]\frac{\partial \beta_{x,y}}{\partial \delta}ds \pm \frac{1}{4\pi}\oint K_2 \beta_{x,y} D^{(2)}_x ds. \quad (4)$$

Here, $\frac{\partial \beta_{x,y,\delta}}{\partial \delta}$ is the off-momentum β-beat given by Eq. (5), $D^{(2)}_x$ is the second order horizontal dispersion function, and $D^{(2)}_x = \frac{\partial D_{x,\delta}}{\partial \delta}|_{\delta=0}$.

The first term in Eq. (4) comes from the changes of magnetic strengths for off-momentum particles, which is smaller than the other terms and can be ignored for most of the time. The second term in Eq. (4) arises from the first order off-momentum β-beat. The third term of Eq. (4) comes from the second order dispersion.

From Eq. (2), the first order off-momentum β-beat is determined by the half-integer RDTs. The horizontal half-integer RDT is defined as

$$h_{20001} = \sum_i^N [-(K_1 L)_i + (K_2 D_x L)_i]\beta_{x,i} e^{-i2\phi_{x,i}}. \quad (5)$$

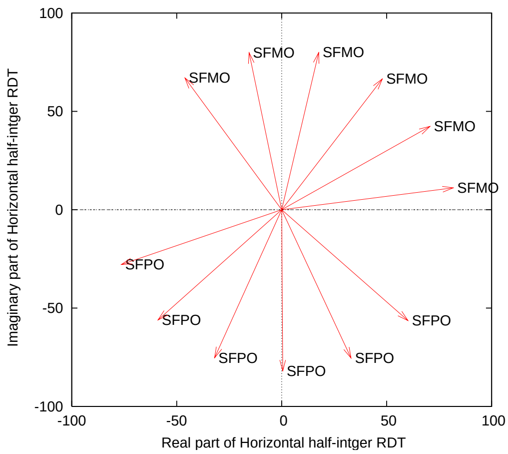

Figure 1: Contributions to horizontal half-integer RDT from all the focusing sextupoles in first arc.

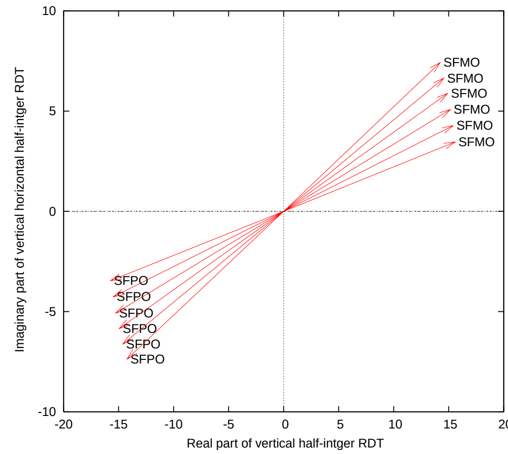

Figure 2: Contributions to vertical half-integer RDT from all the focusing sextupoles in first arc.

The vertical half-integer RDT is defined as

$$h_{00201} = \sum_{i}^{N}[+(K_1L)_i - (K_2D_xL)_i]\beta_{y,i}e^{-i2\phi_{y,i}}. \quad (6)$$

In Eq. (5) and (6) we used summations rather than integrals. The half-integer RDTs h_{20001} and h_{00201} are defined at the starting point of the lattice. Half-integer RDTs h_{20001} and h_{00201} will cause $2Q_x$ and $2Q_y$ betatron resonances respectively. From Eq. (2) and Eq. (4), the first order off-momentum β-beats and the second order chromaticities both are closely related to the half-integer RDTs h_{20001} and h_{00201}.

CALCULATING HALF-INTEGER RDT

Contributions from 12 SFs in one arc

As an example, with the 2009 RHIC 100 GeV polarized proton (pp) run Blue ring lattice, we calculate the contributions from the focusing chromatic sextupoles in the first outer arc from IP6 to IP8. Fig. 1 and Fig. 2 show the contributions of the 12 focusing sextupoles in the first arc between IP6 and IP8 to the horizontal and vertical half-integer RDTs. From Fig. 1 and Fig. 2, according to their contributions to the half-integer RDTs, the 12 focusing chromatic sextupoles in the first arc clearly can be sorted into two groups, that is, SFPO and SFMO. The contributions from the 6 individual sextupoles in each group of SFPO or SFMO have similar contribution amplitudes and angles to the half-integer RDTs.

Also from Fig. 1 and Fig. 2, we note that the contributions to the horizontal or the vertical half-integer RDT from the SFPO and SFDO sextupoles in this arc are almost 180 degree apart. Therefore, if the SFPO and SFDO are powered with the same strengths, their contributions to the half-integer RDTs will cancel. However, if the SFPO and SFDO are powered with opposite strengths, they will give net contributions to the half-integer RDTs.

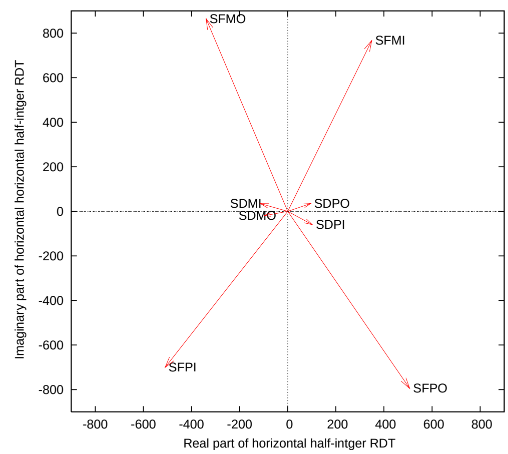

Figure 3: Contributions of all 8 sextupole families to the horizontal half-integer RDT.

4 knobs to minimize half-integer RDTs

In the RHIC, there are 24 independent sextupole power supplies which corresponds to 24 sub-sextupole families. However, in the RHIC control room, they are sorted into 8 families for second order chromaticity correction. The chromatic sextupoles in the 3 outer arcs are sorted as 6*(SFPO, SDMO, SFMO, SDMO), while the chromatic sextupoles in the 3 inner arcs are sorted as 6*(SFPI, SDMI, SFMI, SDMI). Fig. 3 and Fig. 4 shows their contributions to the horizontal and vertical half-integer RDTs.

From Fig. 3 and Fig. 4, we clearly see the opposite contributions to the half-integer RDTs from the 2 families in each pairs (SFPO and SFMO), (SDPO and SDMO), (SFPI and SFMI), and (SDPI and SDMI). For example, for the pair of (SFPO and SFMO), if the SFPO and SFMO families are powered with the same strength their contributions to the half-integer RDTs will cancel. However, if the SFPO and SFMO families are powered oppositely with the same absolute strength, they will give net contributions to the half-integer RDTs. Also from Fig. 3 and Fig. 4, the

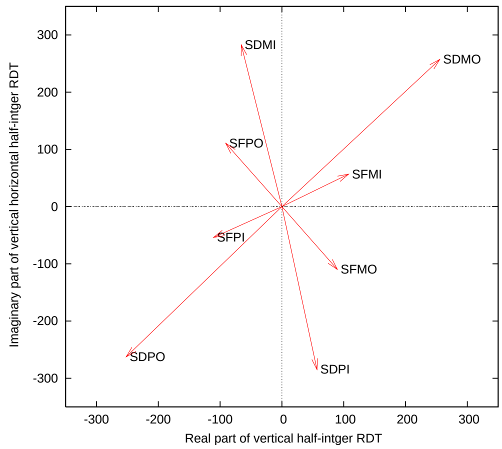

Figure 4: Contributions of all 8 sextupole families to the vertical half-integer RDT.

two pairs (SFPO and SFMO) and (SFPI and SFMI) have larger contributions to the horizontal half-integer RDT, while the two pairs (SDPO and SDMO) and (SDPI and SDMI) have larger contributions to the vertical half-integer RDT. Therefore, adjusting the pair of (SFPO and SFMO) or of (SFPI and SFMI) mostly will affect the the horizontal second order chromaticity and the first order horizontal off-momentum β-beat, while adjusting the pair of (SDPO and SDMO) or of (SDPI and SDMI) mostly will affect the vertical second order chromaticity and the vertical first order vertical off-momentum β-beat.

In summary, there are 4 knobs corresponding to 4 pairs of chromatic sextupole families to easily and effectively minimize half-integer RDTs to correct the second order chromaticities in the RHIC. The 4 knobs are the 4 pairs of chromatic sextupole families (SFPO and SFMO), (SFPI and SFMI) (SDPO and SDMO), and (SDPI and SDMI). In each correction step, we adjust the two families of one pair with same amount of absolute strength but with opposite signs. The first order chromaticity will be kept before and after second order chromaticity correction.

Numeric simulation

Previously, we used 8 families or 8 variables to minimize the second and the third order chromaticities and the first order off-momentum β-beats at IP6 with the Harmon module in the MAD8 [3, 4]. Most of the time, the Harmon module gives very good corrections. However, we notice that the Harmon module some time fails to avoid the reversal of polarity of sextupole families, and introduces a large unbalance in the correction strengths among the 8 families for the low β^* RHIC 2009 run pp lattices.

Here we numerically simulate the second order chromaticity correction with the above 4-knob method. Before the second order chromaticity correction, the first order chromaticities are corrected with 2-family scheme to 1 unit. On top of the 2-family scheme correction, we use the above 4 knobs to correct the second order chromaticities.

Table 1: Dynamic apertures in 10^6 turn trackings

Case	minimum DA in 5 angles [σ]
$\beta^* = 0.9$m lattice:	
$\xi^{(2)}$ corr with Harmon	3.6
$\xi^{(2)}$ corr with 4-knobs	4.5
$\beta^* = 0.7$m lattice:	
$\xi^{(2)}$ corr with Harmon	3.5
$\xi^{(2)}$ corr with 4-knobs	4.1
$\beta^* = 0.5$m lattice:	
$\xi^{(2)}$ corr with Harmon	3.4
$\xi^{(2)}$ corr with 4-knobs	3.5

With the RHIC run09 100GeV pp lattices, we find that the pair (SFPI and SFMI) will significantly affect the horiozontal second order chromatiicty while the pair (SDPO and SDMO) will significantly affect the vertical second order chromatiicty. After a few of iterations, we can easily find the correction strengths to correct the second order chromaticities below 500.

Table 1 shows the calculated 10^6 turn dynamic apertures with the second order chromaticity corrections with the Harmon module and the 4-knob method. Three lattices with $\beta^* = 0.9$m, 0.7m, 0.5m are adopted for this study. From Table 1, the 4-knob method gives larger dynamic apertures than the Harmon module for all the three lattices. And we also notice that the 4-knob method reduces the unbalance in the correction strengths among the sextupole families and avoids reversing sextupole polarities.

CONCLUSION

Based on the contributions of the chromatic sextupole families to the half-integer resonance driving terms in the RHIC rings, we sorted the RHIC chromatic sextupoles into 4 pairs to correct the second order chromaticities. The method will not change the first order chromaticities during knobbing. It reduces the unbalance in the correction strengths among the sextupole families, and avoids reversing sextupole polarities.

REFERENCES

[1] S. Tepikian, *Nonlinear chromaticity correction*, 2006 RHIC Retreat, July 10-13, 2006, Port Jefferson, NY. Available at http://www.c-ad.bnl.gov/RHIC/Retreat2006/agenda.html.

[2] J. Bengtsson, *The sextupole scheme for the Swiss Light Source(SLS): an analytical approach*, SLS Note 9/97, March 7, 1997.

[3] Y. Luo, *Online nonlinear chromaticity correction at RHIC store*, November 2-3, 2006, RHIC 2006 Accelerator Physics Experiment Workshop, November 2-3, 2005, BNL, Upton, NY.

[4] Y. Luo, *Nonlinear chromaticity correction in RHIC run 2007*, 2007 RHIC Retreat, July 16-17, 2007, Foxwood, CT.