ANALYTICAL FORMULA FOR THE TRANSIENT BUNCH LENGTHENING BY A BETATRON MOTION ALONG BENDING SECTIONS

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Abstract

The formulas of longitudinal and transverse coupling for storage rings are valid for any single-pass line. Simple analytical formulas can easily calculate transient state of the longitudinal and transverse coupling. The transient bunch lengthening is expressed by the product of three factors: the square root of horizontal betatron emittance, a betatron phase factor, and the square root of the H-function, in other words, dispersion action. This effect should be considered when a short electron bunch passes through quasi-isochronous bending sections when short-pulsed radiation is used. The formulae can easily calculate the longitudinal displacement of a synchrotron-oscillating particle after many revolutions with finite chromaticity.

INTRODUCTION

Production of an electron bunch as short as a few picoseconds (ps) or femtoseconds (fs) is an important theme in the research field of synchrotron radiation sources. Since the advent of electron storage rings, numerous researchers have strived to realize stable quasi-isochronous operations [1] to store a short electron bunch. These efforts realized the user operation at BESSY [2], while simultaneously discovering the limitation of this method. The storage ring must be highly stable in quasi-isochronous operation and also the stored bunch current is limited to a very low longitudinal instability threshold. The bunch shortening is practically limited to a few ps. On the other hand, some researchers proposed new ideas capable of producing sub-ps bunch, which would overcome the limitation in the storage ring. They are laser bunch slicing [3], generation of a thin tilted bunch [4-6], and a temporal circulation of a short bunch [7, 8]. In these methods the extremely short bunch is produced instantaneously or locally, then it is important to understand the transient process.

The coupling between longitudinal and transverse can be a serious problem in the extreme limit of bunch shortening. Theoretically the coupling is expressed by the H-function, which is the action of the dispersion in the betatron oscillation phase space, introduced by A. Piwinski and A. Wrulich [9] to calculate synchro-beta resonance. Later, the coupling effect at the off-resonant state was calculated for the quasi-isochronous electron storage ring [10, 11]. It is also an issue in the special case of single-pass line with bending arcs [8]. This article demonstrates that the simple analytical formulas obtained for storage ring [10] are valid for any single-pass line. In most proposals of the production of a short bunch, the coupling effect was numerically calculated [3, 8, 12]. On the other hand, the analytical formulas shown in this article are general and easy to be calculated because they contain conventional terminology such as Twiss parameters and the dispersion function. As examples, we will explain applications of the present formula for two methods of transient short bunch production.

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ANALYTICAL FORMULAS

The discussion herein focuses on the linear effect of the horizontal betatron motion. It is assumed that the vertical betatron motion and energy displacement are negligible, and higher order effects of betatron motion are ignored. Coordinates x and s are the displacement from the reference electron in horizontal (radial) direction and the azimuthal coordinate, respectively. Displacement from the reference electron in the s direction is referred by z. Twiss parameters and the betatron phase are denoted as $\alpha(s)$, $\beta(s)$, $\gamma(s)$ and $\psi(s)$.

Formulas for Storage Ring

The change in the path length from the reference in one revolution, δL, is given by

$$\delta L = \int_{s_S}^{s_S+L_0} [\sqrt{\varepsilon_{CSI}\beta(s)} \sin\psi(s) / \rho(s)] ds \,. \qquad (1)$$

Here L_0 is a circumference, s_S is s at the light source point, , ε_{CSI} is Courant Snyder Invariant of a particle and $\rho(s)$ is the curvature of radius of the reference orbit. In additions to the Twiss parameters (α, β, and γ), betatron phase (ψ) and dispersion functions (η and η'), we will use functions $H(s)$ and $\psi_H(s)$ defined by

$$\sqrt{H} \sin\psi_H = \eta(\alpha/\sqrt{\beta}) + \eta'\sqrt{\beta} \,, \qquad (2a)$$

$$\sqrt{H} \cos\psi_H = \eta/\sqrt{\beta} \,, \qquad (2b)$$

Using these functions Eq.(1) can be rewritten as

$$\delta L_n(s_S) = \sqrt{\varepsilon_{CSI} H(s_S)} \{-\cos[2n\pi\nu + \psi(s_S) + \psi_H(s_S)] + \cos[\psi(s_S) + \psi_H(s_S)]\} \,. \qquad (3)$$

The longitudinal displacement from the reference z after n revolutions is expressed as

$$z = -\delta L_n - z_S \,. \qquad (4)$$

Here z_S is the initial displacement, which depends on the initial phase $\psi(s_S)$. If the reference is defined as an average over $\psi(s_S)$ for many revolutions, z_S is determined and the longitudinal displacement $z(s_S)$ is given by

$$z(s_S) = \sqrt{\varepsilon_{CSI} H(s_S)} \cos[2n\pi\nu + \psi(s_S) + \psi_H(s_S)], \qquad (5)$$

where the horizontal displacement $x(s_S)$ is given by

$$x(s_S) = \sqrt{\varepsilon_{CSI}\beta(s_S)}\sin[2n\pi\nu + \psi(s_S)]. \quad (6)$$

When the density distribution of the particles in a bunch in transverse phase space is a Gaussian function with rms emittance of ε_0, the distribution in z axis by the coupling becomes a Gaussian distribution with a standard deviation (=rms) of $\sqrt{\varepsilon H}$.

Formulas for a Single Pass Bending Section

Using the appropriate definition of Twiss parameters and the dispersion function, Eq. (3) can be applied to a single pass transport line with bending sections [13]. We should define one location where z_S in Eq. (4) is zero. We indicate parameters at this special location with suffix '1' whereas parameters at the radiator, in other words, the target location or the light source point, are denoted with suffix '2'. Therefore the displacement of any particle at the radiator is

$$z_2 = \sqrt{\varepsilon_{CSI}H_2}\cos(\Delta\psi_{21} + \psi_1 + \psi_{H2}) - \sqrt{\varepsilon_{CSI}H_1}\cos(\psi_1 + \psi_{H1}). \quad (7)$$

Here $\Delta\psi_{21}$ is the betatron phase advance from the initial to the radiator.

Assuming that the initial phase ψ_1 is uniformly distributed, the average of z_2^2 over ψ_1 can be calculated as

$$\frac{1}{2\pi}\int_0^{2\pi} z_2^2\, d\psi_1 = \frac{\varepsilon_{CSI}}{2}[H_1 + H_2 - 2\sqrt{H_1H_2}\cos(\Delta\psi_{21} + \psi_{H2} - \psi_{H1})]. \quad (8)$$

Therefore, assuming a Gaussian distribution in the transverse phase space, the rms bunch lengthening is expressed by

$$\sigma_{z2} = \sqrt{\varepsilon_0[H_1 + H_2 - 2\sqrt{H_1H_2}\cos(\Delta\psi_{21} + \psi_{H2} - \psi_{H1})]}. \quad (9)$$

EXAMPLES CALCULATIONS

This section provides two example applications of the formulas derived in the previous section.

Laser Bunch Slicing in Quasi-Isochronous Ring

M. Shimada *et al.* observed a coherent synchrotron radiation (CSR) in THz region from a laser bunch sliced beam in a quasi-isochronous ring [12]. The lengthening effect discussed in this article clearly appeared in their experiment because (1) dilution by the energy spread was very small, (2) transverse emittance was considerable, and (3) synchrotron radiation was observed for many revolutions. They measured the CSR power from a dip in a bunch for certain revolutions after slicing. Their results suggested that the length of the dip oscillated according to the betatron tune (ν_x). Theoretically, they confirmed their results by numerical calculations of the transfer coefficients for each revolution. However, Eq. (9) gives much simple explanation to the oscillation of bunch lengthening.

When both of H_1 and H_2 are non-zero, the bunch lengthening oscillates with the betatron oscillation. The maximum and the minimum bunch lengthening are $\sqrt{\varepsilon_0}(\sqrt{H_1} + \sqrt{H_2})$ and $\sqrt{\varepsilon_0}(\sqrt{H_1} - \sqrt{H_2})$, respectively.

Fig.1 shows the results of calculations using Eq.(8). The parameters of the ring at the experiment is listed in Table I.

Table I Parameters of UVSOR-II at the experiment [14].

Betatron tune ν_x	3.66	3.53
Natural emittance ε_0 (nm)	177	139
H_1 / H_2 (m)	0.26 / 0.99	0.12 / 0.89
ψ_{H1} / ψ_{H2}	π / 1.73π	π / 1.68π
$\Delta\psi_{21}$ (1st turn)	1.22π	1.22π

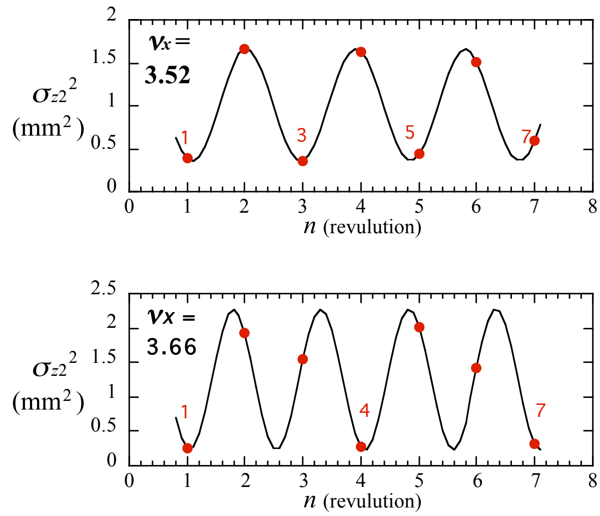

Figure 1: Bunch lengthening by a betatron motion calculated from Eq. (9). In the experiment at UVSOR-II [12] the strong CSR was observed at the 1st and the 3rd turn for ν_x =3.52. The CSR was strong at the 1st and the 4th turn for ν_x=3.66. It was clear that the strong CSR was observed when the bunch lengthening was small. It suggests that if $\Delta\psi_{21}$ were not 1.22π, the variations of CSR power were not clear.

Chromatic Bunch Compression

The author is proposing new method of generating CSR in THz region in the storage ring using chromaticity modulation [15]. When a bunch is transversely deflected in electron storage rings, the bunch would have spatial 'tilt' structure after a half of the synchrotron oscillation period. This structure was produced by a betatron phase difference from the bunch head to the bunch tail. W. Guo *et al.* showed that this "vertical" tilt can be used to generate short pulsed X-ray [16]. On the other hand, when the beam is "horizontally" deflected, the longitudinal and transverse coupling produces a density modulation in a bunch. This modulation can be a source of CSR.

In this report we will calculate more simple case, where the chromaticity is a constant with time. At this situation the injected beam would be temporary compressed in the storage ring. The horizontal betatron phase shift $\Delta\psi_x$ from the bunch centre is given as

$$\Delta\psi_x = -(2\omega_0/\alpha_p)\xi_x\tau. \quad (10)$$

Here ω_0 is the angular frequency of the revolution, α_p is the momentum compaction factor, ξ_x is the horizontal chromaticity, and τ is the timing displacement at the rf cavity. Notice, Eq.(5) and Eq.(6) show that the phase difference between the longitudinal and the horizontal oscillation is $\psi_H + \pi$. At the timing when $2n\pi\nu + \psi(s_S) = 0$ at the bunch centre, the longitudinal displacement by the coupling is given by

$$z = \sqrt{\varepsilon_{CSI} H} \cos\left[\psi_H - \left(2\omega_0 / \alpha_p\right)\xi_x \tau\right]. \quad (11)$$

This gives a density modulation for large ξ_x and a bunch shape deformation for small ξ_x.

Fig. 2 shows an example of the calculation using Eq.(11). The deformed bunch shape of the injected beam after a half of the synchrotron oscillation period at the NewSUBARU storage ring [7]. The assumed observation point was the beam diagnostics port named SR3. The parameters were, ε_{CSI}=1.5 π μm, β_x=0.82m, H=0.37m, ψ_H=-0.15π, $\omega_0/2\pi$=2525 kHz, α_p=0.0013, ξ_x =6 (ξ_x =3 in the real machine), and the bunch length was 20 ps (full width). The peak bunch density was about 60% larger than that at the injection. The calculation was not a tracking simulation, then we ignored the non-linear effect and the perturbation by the wake-field and CSR. We also ignored the spread by the horizontal emittance (ε_0≈0.04 π μm at the real machine).

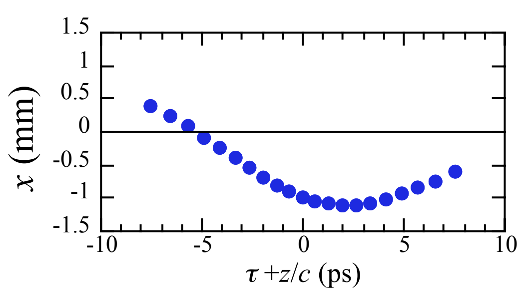

Figure 2: Bunch shape of the injected beam at a half synchrotron oscillation period after the injection. The 21 points indicate electrons, which had been equally spaced by 1 ps at the injection. The appropriate betatron phase was chosen so that the density enhancement took place at the bunch centre. The horizontal chromaticity was set at twice of the normal operation value.

SUMMARY

The simple analytical formula for the longitudinal and transverse coupling is explained. This analytical expression is helpful to catch a rough image at various situations. At the explained two situations the calculation of the beam behavior is much easier using the analytical form than using the transfer matrix.

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