# A NON INVASIVE TECHNIQUE FOR THE TRANSVERSE MATCHING IN A PERIODIC FOCUSING CHANNEL OF A LINAC

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## Abstract

A main interest in the high intensity ion linacs is the control of the particle loss in the vacuum chamber. A extremely low fraction of the beam, from  $10^{-7}$  up to  $10^{-4}$ , is sufficient to complicate the hands on maintenance in such accelerator. Beam mismatching being a major source of halo, it is proposed a non invasive technique to adapt the beam to a periodic focusing channel of a linac based on a FDO of FODO lattice. It is demonstrated that only the matched beam can correspond to a particular signature of the quadrupolar moment of the Beam Positions Monitors. This technique allows also to measure the emittance value or evolution along the channel.

### **INTRODUCTION**

High power ion linacs have become increasingly attractive in recent years. Among the possible applications are heavy ion drivers for thermonuclear energy or rare ion beam production, transmutation of radioactive wastes and the spallation sources of neutrons for matter research. High intensity charged particle beams can develop extended low density halos. The existence of halos can have serious consequences for the hands on maintenance. Studies show that emittance growth arising from beam mismatch manifests itself in the form of a halo [1]. At J-PARC, in each focusing period of the Separate Drift Tube Linac (SDTL) section, wire-scanners are used to math the beam profile [2]. This device is limited in terms of beam pulse length as it intersects the beam and can not be used for any nominal set-up. This thermal problem has been solved at SNS by using a laser profiler [3]. The electron photodetachment of the H<sup>-</sup> particle makes it is possible to obtain an image of the transverse density without material in the beam. One drawback of this technique is that it is not usable for naked nucleus beams (proton or deuteron for instance). This restriction applies for several high power protons linacs [4, 5, 6].

The use of the quadrupolar moment of the Beam Position Monitors (BPMs) have been proposed by Miller at SLAC [7] to determine the emittance and the Twiss parameters [9] at the entrance of a channel with the help of six pick-ups installed along the linac. The Miller method has been adapted to rings by Jansson [8]. The use of the quadrupole pick-ups in rings has also focused on determining the phase and the amplitude of the betatron and revolution frequencies in the raw pick-up [10]. In this note, it is proposed to use the quadrupolar moment of the BPMs to adapt the beam transversely to a periodic channel of a linac. It is developed the idea that the non invasive measurement of the BPMs can be used to match the transverse beam envelop if, at least, one BPM is located in one of the quadrupoles of the lattice (FD0 or FODO, F for Focusing, D Defocusing and O represents a drift space). This technique is then relevant for the naked nucleus beams as well as the other ions. It is also cost effective as the BPMs are required anyway for the orbit control. We will also see that the proposed technique can be used to monitor the emittance value or evolution.

#### MODEL

The quadrupolar moment is proportional to the difference of the squares of the two transverse beam sizes  $\sigma_{x,y}$ [7] and, then, is a function of the betatron function  $\beta_{x,y}$ and the emittances  $\varepsilon_{x,y}$ . Taking into account the property  $\varepsilon_x = \varepsilon_y = \varepsilon$  which occurs in intense ion beam linac because of the space charge which strongly couples the two transverse planes, the quadrupolar signal is proportional to:

$$M_Q \propto \sigma_x^2 - \sigma_y^2 = \varepsilon_x \beta_x - \varepsilon_y \beta_y \propto \beta_x - \beta_y \qquad (1)$$

Here, we assumed that the section of the linac is non dispersive and that the contribution of the centres ( $\propto \bar{x}^2$  or  $\bar{y}^2$ ) to the quadrupolar signal can be removed with the help of the dipolar signals which are used to control the orbit. A first property can be obtained from this previous equation, if  $M_Q(s_1) = M_Q(s_2)$  where  $s_1$  and  $s_2$  corresponds to two different locations in the machine, we find that:

$$\beta_x(s_1) - \beta_x(s_2) = \beta_y(s_1) - \beta_y(s_2) = \Delta \qquad (2)$$

Let us consider a FODO or FDO focusing period architecture. The linac section is an array of this lattice which can be arbitrary started with a focusing quadrupole in the horizontal plane. The drift spaces between the quadrupoles are used to insert pumping or accelerating system and can correspond to any length whatever. The transfer matrix T of this sector which transforms the position and angle  $(x_1 = \sqrt{\beta_1 \varepsilon}, x'_1 = \sqrt{\frac{1+\alpha_1^2}{\beta_1}\varepsilon})$  at the entrance  $s_1$  to a new set of position and angle  $(x_2 = \sqrt{\beta_2 \varepsilon}, x'_2 = \sqrt{\frac{1+\alpha_2^2}{\beta_2}\varepsilon})$  at the exit  $s_2$  can be written [9]:

$$\begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\delta + \alpha_1 \sin\delta) & \sqrt{\beta_1\beta_2}\sin\delta \\ \frac{(\alpha_1 - \alpha_2)\cos\delta - (1 + \alpha_2\alpha_1)\sin\delta}{\sqrt{\beta_1\beta_2}} & \sqrt{\frac{\beta_1}{\beta_2}}(\cos\delta - \alpha_2\sin\delta) \end{pmatrix}$$
(3)

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where  $\delta$  is the phase advance from  $s_1$  to  $s_2$ . For the horizontal plane, as  $s_1$  corresponds to the focusing thin lens location (or lens center), we have the property  $\alpha_1 = \alpha_2 =$ 0 (the beam which enters in the lens is diverging and it is converging when it gets out). The matrix T is then reduced to:

$$\begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}\cos\delta & \sqrt{\beta_1\beta_2}\sin\delta \\ -\frac{1}{\sqrt{\beta_1\beta_2}}\sin\delta & \sqrt{\frac{\beta_1}{\beta_2}}\cos\delta \end{pmatrix} = \begin{pmatrix} C_{12} & S_{12} \\ C'_{12} & S'_{12} \end{pmatrix}$$
(4)

By applying this transfer matrix, we find:

$$\beta_2 = C_{12}^2 \beta_1 + S_{12}^2 \frac{1}{\beta_1} \tag{5}$$

The parameter  $\Delta$  can then be calculated :

$$\Delta_{12} = \beta_2 - \beta_1 = \left(C_{12}^2 - 1\right)\beta_1 + S_{12}^2 \frac{1}{\beta_1} \tag{6}$$

We can repeat the procedure for the following period from  $s_2$  to  $s_3$  and find a similar expression  $\Delta_{23} = f(\beta_3, \beta_2, C_{12}, S_{12})$ . If we ask for  $\Delta_{12} = \Delta_{23}$ , we find that  $\beta_2$  and  $\beta_1$  are mathematically linked by two possible relations: either  $\beta_2 = \beta_1$  or  $\beta_2 = S_{12}^2/\beta_1(C_{12}^2 - 1)$ . The second solution is equivalent to the first one if  $\Delta_{23} = \Delta_{34}$ . Only one case is then possible and it corresponds to  $\Delta_{(i)(i+1)} = 0$ .

Let us check that we got also the property  $\alpha_y(s_1) = \alpha_y(s_2)$ . Taking into account that  $\beta_y(s_1) = \beta_y(s_2)$ , the matrix 3 can be rewritten for the vertical plane and the parameter  $\alpha_y(s_2) = \alpha_2$  is equal to:

$$-P_{12}P_{12}'\beta_1 + [P_{12}Q_{12}' + Q_{12}P_{12}']\alpha_1 - Q_{12}'Q_{12}\frac{1+\alpha_1^2}{\beta_1}$$
(7)

with  $P_{12} = cos\delta + \alpha_1 sin\delta$ ,  $Q_{12} = \beta_1 sin\delta$ ,  $P'_{12} = ](\alpha_1 - \alpha_2)cos\delta - (1 + \alpha_2\alpha_1)sin\delta]/\beta_1$  and  $Q'_{12} = cos\delta - \alpha_2 sin\delta$ . By replacing the parameters  $P_{12}$ ,  $P'_{12}$ ,  $Q_{12}$ and  $Q'_{12}$  by these previous expressions, one finds  $\alpha_2 = \alpha_1$ . With this last condition, we have verified that once  $M_Q$  is constant (entrance and exit of two consecutive focusing periods), only the matched beam is solution of the system if the  $M_Q$  is measured at the center of one lens. In case the BPM is located between the two lenses, the parameters  $\alpha$ are unknown and there is an infinite number of solutions for the betatron function which obey to the equation 2 with the condition  $\Delta$  is equal to zero.

# NON CONSTANT CHANNEL AND EMITTANCE MONITORING

We assumed that a matched beam corresponds to a beam which has similar parameters before and after the focusing period. In many real cases, this definition is not useful because the parameters of the focusing periods change slowly in the machine (energy variation, design choices). Nevertheless, a very close approach is often used to define the



Figure 1: Shift between reference evolutions and measurement of the  $M_Q$ .

adapted beam: the matched conditions are assumed to be performed when the beam parameters vary very slowly too and that its envelop beating has the periodicity of the channel. These matched parameters are anyway very close to the ones previously defined.

To match the beam with the help of the quadrupolar moment of the BPMs in lenses of the focusing period, there are several techniques. In case the channel is almost constant, the simple condition  $M_Q$  is a constant is enough. For example, to tune the beam at the entrance of the channel we can minimize the sum of the square of the  $M_Q$  variations period after period.

In case, the channel varies significantly, one possible technique may be based on a a priori knowledge of the emittance evolution or its value. The idea is to minimize the shift between a reference evolution of the  $M_Q$  and its measurement. The reference has to be build with the known parameters of the channel (focusing strength, energy, peak current) and this a priori knowledge of the emittance. One may think this a priori knowledge is the weakness of this technique but it is exactly the opposite. By varying this reference, the matching procedure offers the possibility to measure the real emittance value or evolution. Indeed, the previous defined shift is minimum when the theoretical reference corresponds to the values of the machine. This is illustrated in the figure 1 in which it is plotted this shift when different emittances are used to build the reference  $M_Q$  law. For each point, the minimization is started with a mismatched beam (matched Twiss parameters are changed by a 30% factor) and it is stopped after the same number of iterations.

### **BENCHMARK**

To benchmark the technique in case of more realisitic conditions, it is proposed to use a section of a high power linac based on a FDO lattice, the drift being used to include a three mono gap resonators to accelerate from 180 MeV up to 520 MeV (eleven focusing periods). A proton with a peak current of 75 mA beam is transported in or-

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der to be in a significant space charge regime. The benchmark consists in evaluating several beam envelop tuning procedures in presence of unperfect focusing elements. Defaults for the quadrupoles (gradient errors between  $\pm 1\%$ ) and the cavities (amplitude between  $\pm 1\%$  and phase between  $\pm 1$ degree) have been randomly generated in order to simulate 2000 different linacs. A mismatched beam is assumed at the entrance of the section: Twiss parameters are modified to be 30% away from their nominal values.

Several tuning techniques are compared. For each technique, the tuning is performed with the first four quadrupoles of the section. Only the optimization criteria is changed. The first technique is a minimization of the difference between the measured size (it is assumed that the use of transverse beam profiler are possible) and a reference size for each focusing period. The second technique is a minimization of the difference between the measured size at one focusing period and the measured size for the following focusing period. A smooth evolution of the beam envelope is then obtained. For both techniques, the beam profiler is located between the two quadrupoles at each focusing period. The third technique is a minimization of the measured quadrupolar moment with a quadrupolar pickup located between the two quadrupoles at each focusing period. The fourth technique is a minimization of the difference between the measured quadrupolar moment and a reference quadrupolar moment for each focusing period. The quadrupolar pickup is located inside the first quadrupole at each focusing period. To illustrate the necessity of a correction procedure, a statistical study has been also performed without any tuning procedure. The beam is simply transported in each generated linac. To tend to a more realistic benchmark, an error is also assumed for the measured signals. Their amplitude is typically about 10%. All simulations have been performed with the TraceWin code [11] using the simplex optimization. The figure 2 shows the statistical distribution of the transverse halo parameter [12] for the beam at the exit of the simulated linacs. First, this figure illustrates the importance of the beam matching. For a few linacs, the halo extension can be extremely increased. Second, the technique proposed in this paper appears to be equivalent to a direct measurement of the beam size. In other words, to know the size of the beam doesn't allow to reach a better efficiency for the beam matching than the use of the quadrupolar signal. Third, the uselessness of the quadrupolar signal when it is measured between quadrupoles can be also noticed.

### SUMMARY

The technique developped in this paper allows to perform the transverse matching of a beam in a periodic section of a linac (FDO or FODO). This matching technique based on the use of the quadrupolar signal of BPMs is non invasive and cost effective as the BPMs are mandatory to control the orbit . The interest in this matching procedure can be particularly enhanced by noticing the possibility to



Figure 2: Statistic distribution of the halo parameter at the linac exit for the five techniques.

measure and monitor the emittance value or evolution in the channel. Experimental benchmarks of this technique are planned with the super conducting ion linac under construction at GANIL [5].

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