# DEFINITION OF FOCUSING SYSTEM PARAMETERS ON THE BASIS OF THE ANALYSIS OF A TRANSVERSE BUNCH DYNAMICS IN DIELECTRIC LOADED WAKEFIELD ACCELERATOR \*

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### Abstract

The alternating-sign focusing of high current relativistic electron beams in multi-bunch wakefield acceleration is investigated. These beams are used for generating wake fields in dielectric loaded accelerating structures. We consider ramped charge distribution in the sequence of high current drive bunch. It is shown that the beam focusing system mitigates beam break-up effect and elongates of a maximum distance that the high current beam can travel along the structure. That, in turns, determines the effectiveness of the energy transfer to the accelerated electron beam correspondingly. The optimal parameters of the focusing system on the basis of selfconsistent transverse dynamics analysis are presented.

## MULTIBUNCH WAKEFIELD ACCELERATION SCHEME

The technology of dielectric wakefield electron accelerators is one of the most promising directions in the development of high-field-gradient structures for modern linear accelerators, and it has been extensively studied in recent years [1–4]. The main element of such an accelerating structure is a cylindrical metal waveguide filled with a dielectric and having an axial vacuum channel. A short (1–4 mm) driving electron bunch possessing a large charge (20–100 nC) travelling along the channel generates a  $TM_{01}$  wakefield mode of the Cherenkov radiation with a longitudinal electric field component. The subsequent high-energy witness (driven) electron bunch with a small charge following the former bunch at a certain delay selected to fit the accelerating phase of the wakefield, is to be accelerated by this field.

The driving bunch charge, which can be passed through waveguide, is limited by technical capacities of photo injector, Coulomb field created by bunch spatial charge and vacuum channel waveguide size. To increase in accelerating field it is used bunches chain instead of wave-guide single bunch. Flat Bunch Train (FBT) accelerating scheme with a flat profile bunches sequence is used for coherent addition fields bunches and increase overall acceleration fields.

Increasing accelerator efficiency is connected with increasing of a part of driving bunch energy transferred to accelerated particles. The energy transformation ratio R is defined as the ratio of the maximum electron energy increment in the witness bunch to the maximum electron

energy loss in the driving bunch  $R = \Delta W^+ / \max |\Delta W^-|$ . In order to increase for R, a train of Gaussian bunches with a period of d having a triangular envelope (Ramped Bunch Train, RBT) is used. The charge in this train varies from a minimum for the first bunch to a maximum for the last bunch, the four leading bunches in the train are subjected to the action of a relatively low retarding field of the same amplitude.

In this case it is necessary both full extraction of energy from the driving bunches  $\Delta W^-$ , and increasing passed for witness bunch energy  $\Delta W^+$ . This criterion requires ensuring the maximum possible flight of *L* which beam interacts with accelerating structure, which in turn makes substantial demands lateral stability of the accelerating and accelerated beams.

An excess deviation caused by the deflecting forces results in the electrons of bunches fall to the waveguide wall and do not manage to transmit their energy to wake field and further acceleration becomes impossible. Therefore, the transverse instabilities restrict the possibility of gaining energy from the train of driving bunches.

An analysis of the transverse dynamics for a train of high-current bunches shows that a radial deviation of the beam poses limitations on the distance L travelled by bunches in the waveguide [4]. Focusing only partially suppresses the lateral instabilities, leading to a relative increase of the flying range in comparison with the case without the use of the external focusing system.

## **BEAM DYNAMICS EQUATIONS**

The fields calculation we will carry out under the assumption Gaussian charge distribution in bunches both on longitudinal f(z) and on radial f(r) coordinates. Since into the deflection field with the small deviations of beam from the axis the greatest contribution introduces the first azimuthal mode, the force, which acts on the charges in the radial direction, depends on r linearly ( $I_1(kr) \approx kr$  with small kr), integration of the elementary sources of transverse force for the radial coordinate gives the average of Gaussian distribution.

Thus, for radial dynamics calculating it is possible to consider that the charge is concentrated in the center line of the transverse distribution of bunch [4]. Subsequently we will examine the filamentary electronic beam with the longitudinal charge distribution  $f(\zeta)$ , which moves along

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the axis of waveguide with the displacement  $r(\zeta, t)$ . Let us take initial displacement of beam  $r(\zeta, t)|_{t=0} = r_0$ .

The field of Gaussian bunch with charge q can be determined by taking the integral convolution of point charge field with the distribution of charge in a bunch:

$$E_{zli} = \sum \psi_{E_{zn,m}} I_n (k_{rn,m} r(\zeta, t)) \times$$

$$\times \int_0^{\zeta} f(\zeta_0) \cos(k_{zn,m} (\zeta - \zeta_0 - z_i)) I_n (k_{rn,m} r(\zeta_0, t)) d\zeta_0,$$

$$F_{rli} = F_f - e \sum \psi_{F_{rn,m}} I'_n (k_{rn,m} r(\zeta, t)) \times$$

$$\times \int_0^{\zeta} f(\zeta_0) \sin(k_{zn,m} (\zeta - \zeta_0 - z_i)) I_n (k_{rn,m} r(\zeta_0, t)) d\zeta_0,$$
where  $\psi_{E_{zn,m}}$  - are factors of a number, depending on the geometry, dielectric permittivity waveguide and starting charge position,  $\zeta = z - v_z t$ ,  $k_{zn,m} = 2\pi v_{n,m} / (\beta c)$  is longitudinal component of wakefield wave vector,  $v_{n,m}$  are waveguide own frequency, depending on its geometrical and dielectric properties of material,  $z_i$  is the

coordinate of the center of the *i*-th bunch,  $F_f$  is a focusing force. Longitudinal electric field and radial strength created

by bunch train can be found by summing the fields generated by bunches:  $E_z = \sum q_i E_{z1i}$ ,  $F_r = \sum q_i F_{r1i}$ .

The equations of longitudinal and radial dynamics with the relativistic speed of beam in the vacuum are [3]:

$$F_{z} = eE_{z} = m_{e} \frac{d(v_{z}\gamma)}{dt}, \ F_{r} = e(E_{r} + v_{z}B_{\theta}) = m_{e} \frac{d(v_{r}\gamma)}{dt}$$

Longitudinal and radial forces are obtained by integrating the function, which describes the field of emission at point z, r from the point charge, which is located at point with the coordinates  $z_0$ ,  $r_0$ , of that convoluted with the function of the longitudinal bunch charge distribution.

Integrating the equations of longitudinal and radial dynamics for the time and solving the obtained system relative to longitudinal and radial velocity, we will obtain:

$$\begin{split} \beta_z(\zeta) &= \frac{\xi(\zeta)}{\sqrt{1 + \xi(\zeta)^2 + \eta(\zeta)^2}} \,; \, \beta_r(\zeta) = \frac{\eta(\zeta)}{\sqrt{1 + \xi(\zeta)^2 + \eta(\zeta)^2}} \,, \\ \text{where } \xi(\zeta) &= a_z(\zeta)t + \beta_{z0}\gamma_0 \,; \, \eta(\zeta) = a_r(\zeta)t + \beta_{r0}\gamma_0 \,; \\ a_z(\zeta) &= eE_z(\zeta)/(m_ec) \,; \, a_r(\zeta) = F_r(\zeta)/(m_ec) \,; \, \beta = v/c \,. \end{split}$$

After producing repeated integration, we will obtain the dependence of the radial displacement of the centre line of beam on the axis of waveguide from the position of particle in the bunch  $r(\zeta, t)$ .

Significant amplitude of own rejecting fields generated by high current bunches affecting his tail, emphasize focusing system. To keep the high current beam is appropriate to use a rigid focusing system based on FODO focusing [7, 8].

The period focusing system radial force we will approximate the harmonic dependence for taking part a potential "sagging" between quadruple lenses:

$$F_f = -k(l)r = -ec\frac{\partial B(l)}{\partial r}r = -ecr\frac{B_0}{2R_w}\cos\frac{2\pi l}{L_s}.$$

## SIMULATION RESULTS ANALYSIS

Calculations were performed for the waveguide with the parameters: outer waveguide radius  $R_w = 0.6342$  cm, inner radius  $R_c = 0.5$  cm, dielectric constant  $\varepsilon = 16$ , base frequency f = 13.625 GHz, which correspond to the parameters of the test accelerator of the Argon National Laboratory of the USA [2]. The beam consists of four driving bunches and one witness bunch. Beam parameters are: summary charge  $Q_{\Sigma} = -40$ nC, a mean-square length  $\sigma_z = 0.2$  cm and starting energy  $W_d = 30$  MeV. The initial deviation of beam from the axis of waveguide accepted by identical and composes  $r_0 = 0.01$ cm.

In the course of the computer experiment optimal parameters of focusing system: length focusing sections  $L_s$  (Fig. 1, 2) and field induction *B* (Fig. 3, 4) are determined.



Figure 1: FBT acceleration scheme.



Figure 2: RBT acceleration scheme.

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#### **A15 High Intensity Accelerators**

By numerical simulations were received the flying ranges of the system of the bunches before the contact of the wall of waveguide L from length focusing sections Ls for FBT (Fig. 1) and for profiled bunch train RBT (Fig. 2), with a maximum size of focusing field B = 1 T, which corresponds to magnetic field gradient dB/dr = 0.8 T/cm.

For maximum range at a fixed magnetic field value is necessary to match the length section focusing system. The dependencies of increment bunch energy and the flying range vs. the length of the sections are similar. To design a focusing system appropriate to use a wider second resonance.



Figure 3: FBT acceleration scheme.



Figure 4: RBT acceleration scheme.

Dependency of bunch flying range L vs. induction B for FBT and RBT are represented in Fig. 3, 4, at length sections  $L_s = 8.5$  cm.

The figures show stepwise increase flying range and energy before maximum and smooth, slight fall-off after him. Thus, when fixed length sections focusing system, with the induction field is less than some threshold hold beam is impossible. For maximum range it is necessary to match the induction of magnetic field focusing system. Flight range close to maximum is reached, since the threshold induction.

Transmitted bunch energy, in the case of profiled bunch train, is significantly higher than in the case of equal charge bunch train.

Thus, for maximum energy transmission to accelerated bunch need optimal values focusing field and the period length of the focusing element with the specified energy beam sizes and charge.

#### **SUMMARY**

For any type of the wakefield accelerator the length of the structure should be close to a maximally possible length of the effective energy extraction from the drive bunches and the transfer of this energy to the accelerating beam. The focusing system parameters for drive beam formulated in this paper present the conditions to the effective length of the beam pass in the structure under consideration.

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