LONGITUDINAL BUNCH PROFILE DIAGNOSTICS IN THE 50fs RANGE USING COHERENT SMITH-PURCELL RADIATION

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Abstract

We have considered the possibility of using coherent Smith-Purcell radiation for the single-shot determination of the longitudinal profile of 50 fs (FWHM) long electron bunches. A set of three gratings, with periods of 15, 85 and 500 μm, will produce detectable energy in the wavelength region 10-1000μm, which should be adequate for the reconstruction of the bunch shape by the Kramers-Kronig technique. For bunch charges of 10^9 electrons, or more, the radiated energy can be detected by room temperature pyroelectric detectors.

INTRODUCTION

Recent advances in Laser Wakefield Acceleration (LWA) [1] have brought closer the realisation of compact electron accelerators, with electric field gradients (10-100 GV/m). One of the most interesting characteristics of the electron bunches produced by LWA is their very short bunch length which is lies in the few femtosecond range.

Because of the low repetition rate of the laser and the significant variations from bunch to bunch, it is necessary to develop novel electron beam diagnostic techniques capable of measuring the main beam parameters (e.g. the time or 'longitudinal' bunch profile) in a single shot. The determination of the time profile of fs-long bunches can, in principle, be achieved by the use of a coherent radiative process, whereby the bunch is made to emit a small amount of electromagnetic radiation and its time profile is then reconstructed from the measurement of the spectral distribution of this radiation. Radiative processes such as Smith-Purcell (SP) radiation [2] are relatively simple to implement but are not 'direct' processes in the sense that the time profile of the bunch has to be reconstructed from the measured spectral distribution of the radiation. In the present paper we consider the possibility of using coherent SP radiation for the single-shot determination of the time profile of 50 fs long electron bunches. The details of the analysis can be found in [3] and the references cited there.

OVERVIEW OF SP RADIATION AS A DIAGNOSTIC TOOL.

Basic considerations.

The term SP radiation is used to describe the radiation emitted from the surface of a periodic metallic structure (a grating) when a charged particle beam is travelling past the grating. The radiation can be considered as originating from the acceleration of a distribution of charges, the distribution in question being that induced on the grating surface by the travelling electron. This is the 'surface current' (SC) model and is the one followed in this paper. Experiments carried out at energies from a few MeV to 28.5 GeV appear to be in agreement with this theory [4]. Although any charged particle beam will give rise to this radiation, it is assumed that the particles are electrons and that they are highly relativistic ($\beta \simeq 1$). The grating acts as a dispersive element and the wavelength of the radiation will depend on the angle of observation (θ), relative to the beam direction. In most experimental situations the azimuthal angle (ϕ) is close to the vertical, in which case the relationship between wavelength (λ), grating period (*l*) and θ is given by the well-known formula:

$$\lambda - \frac{1}{n}(\frac{1}{\beta} - \cos\theta) \quad (1)$$

where n is the order of the radiation. It is assumed throughout this paper that n=1. The range of emitted wavelengths is determined by the choice of grating period and is, thus, a parameter that can be chosen by the experimenter. A schematic of the process and the axis convention are shown in Fig. 1.

Determination of the time profile

The energy per solid angle generated by the passage of a single electron over a grating of length Z and period *l* is given by the following semi-analytical expression, in CGS units:

$$\left(\frac{dI}{d\Omega}\right)_l = 2\pi e^2 \frac{Z}{l^2}\frac{n^2\beta^3}{(1-\beta\cos\theta)^3)} exp\left[-\frac{2x_0}{\lambda_e}R^2\right] \quad (2)$$

where x_0 is the height of the electron above the grating surface and λ_e is the evanescent wavelength, which is a measure of the coupling efficiency between the electron and the grating and is given by:

$$\lambda_e = \frac{\beta\gamma\lambda}{2\pi\sqrt{1+\beta^2\gamma^2\sin^2\theta\sin^2\phi}} \quad (3)$$

The dimensionless quantity R^2 is the grating efficiency factor, which is a complicated function of the blaze angle of the grating and is calculated numerically. Expression 2 is valid if the width of the grating can be assumed to be

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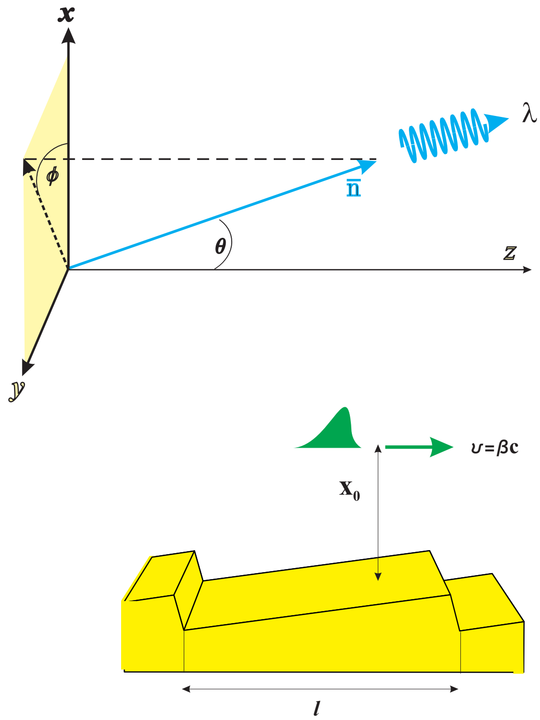

Figure 1: Schematic of the SP radiation and definition of the coordinate system.

infinite, in which case the dependence on x_0 is explicit and appears as an exponential factor. Otherwise, it is necessary to resort to numerical integrations over the finite width *w* of the grating.

If the wavelength is approximately equal to or greater than the bunch length, then the radiation is coherent. In this case, the radiated energy is enhanced significantly and becomes proportional to the square of the number of electrons in the bunch. For a bunch of N_e electrons and having a time profile given by T(t) the radiated energy is given by:

$$\left(\frac{dI}{d\Omega}\right)_{N_e} \sim \left(\frac{dI}{d\Omega}\right)_l N_e^2 \left|\int_{-\inf}^{+\inf} T e^{-i\omega t} dt\right|^2 \quad (4)$$

Therefore, it is possible to shift the SP radiation into the coherent regime through a suitable choice of the grating period. Assuming that the spectral distribution of the coherent SP radiation has been measured, it is then possible to reconstruct the time profile T(t) of the bunch that gave rise to this distribution. This is usually done by applying the Kramers-Kronig (KK) analysis technique for retrieval of the minimum phase of the bunch. A detector array capable of detecting the radiation over a broad angular wavelength range is an essential requirement. SP radiation is particularly advantageous in this respect because it is possible to achieve this wide coverage of wavelengths through the simultaneous use of more than one grating, each having a different periodicity.

Relativistic factor (γ)	1000
Electrons per bunch (N_e)	10^9
Transverse size (σ_x,σ_y)	0.02 mm
Beam position (x_0)	1.0 mm
Bunch time profile	Symmetric Gaussian
Bunch length (FWHM)	50 fs= 15 μm

Table 1: Parameters used for the simulations

Results of simulations.

We have based our study [3] on the assumption that the experimental arrangement can accommodate three gratings. The basic set of beam parameters are listed in Table 1.

The optimum grating periods are determined by the need to ensure: (a) adequate radiated energy (b) the achievement of the widest possible wavelength coverage in order to aid the bunch shape reconstruction and (c) the ability to discriminate between different bunch shapes. The definition of 'adequate' will depend on the choice of detector(s). For the case of a 50 fs bunch (FWHM), the three grating periodicities that were found to give a reasonable compromise between these requirements were l=15, 85 and 500 μm with blaze angles of 30^o, 30^o and 35^o, respectively. The expected angular distribution of SP radiation from these gratings is plotted with solid lines in Figure 2, which shows the differential energy (Joules per steradian per cm of grating length) as a function of wavelength (λ). The wavelength coverage achieved by this combination of gratings is 10-1000 μm, approximately, which is satisfactory.

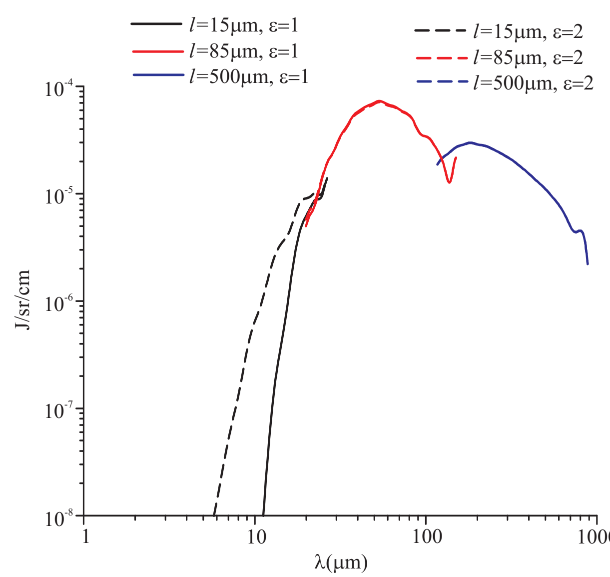

Figure 2: Calculated spectral distributions from two different bunch shapes, both 50fs (FWHM) long. The solid lines are for a symmetric ($\epsilon = 1$) Gaussian bunch and the dashed ones for an asymmetric bunch ($\epsilon = 2$) See text for details.

The above calculations have been made on the assumption that the longitudinal (time) profile of the bunch is a simple, symmetric Gaussian shape. Also shown (with dashed lines) are the spectral distributions that will be radiated if the shape of the bunch is an asymmetric Gaussian, with an asymmetry factor $\epsilon = 2$, which means that the trailing part of the bunch profile has a value of σ that is twice as long as that of the leading edge; the FWHM of the bunch is unchanged, at 50 fs. The change is negligible for the two longer period gratings but it is evident that for good bunch shape discrimination the information provided by the short period grating is important.

The effects of the variation of the beam height (x_0) above the grating surface are shown in Fig. 3, normalised to $x_0 = 1.0mm$. The simulation has been carried out for an observation angle of $\theta = 90^0$ where the wavelength is equal to the period of the grating. As expected, the energy decreases almost exponentially with increasing x_0. This is more pronounced for shorter wavelengths, where good coupling with the grating requires the beam to be close to the grating surface. A choice of $x_0 = 1.0mm$ would be a reasonable compromise between having adequate signal while avoiding the beam halo.

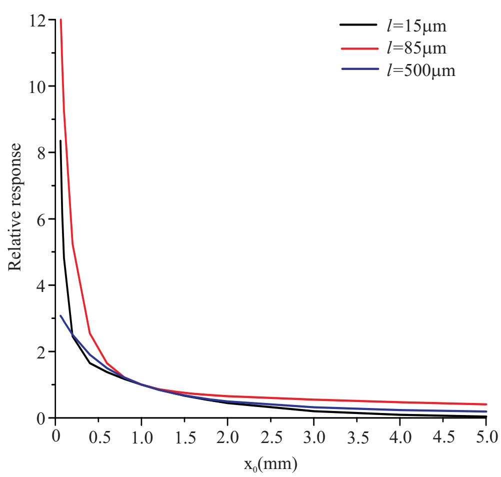

Figure 3: Calculated effect of beam position variation on the radiated energy (see text for details). The output has been normalised to the value expected at x0=1.0mm.

We have carried out simulations for γ values ranging from $\gamma = 1000$ ($E \simeq 500$ MeV) down to $\gamma = 20$ ($E \simeq 10$ MeV). The results indicate that a change in the value of γ from 1000 down to 500 does not have a significant effect on the level of SP radiation. However, further reductions in γ cause a drastic drop in the SP energy.

EXPERIMENTAL ISSUES.

The main items for consideration on the experimental side are detector choice, optical filters and the availability of suitable gratings. The discussion is based on recent experience at SLAC [4], where an array of 11 pyroelectric (PE) detectors was used to detect SP radiation in the wavelength region 0.5-2.6 mm, approximately. The arrangement envisaged in this paper covers a wider part of the spectrum and if one were to exclude He-cooled detectors because of cost and complexity, then this would leave the PE as the only detector covering the whole range of wavelengths. This is a compact, room-temperature and inexpensive device but it has a rather poor noise equivalent power (NEP). One possible alternative, covering wavelengths up to about 20 μm, is the HgCdTe photodiode, which operates at liquid nitrogen temperature and with a NEP about two orders of magnitude better than that of the PE one.

The use of filters in order to discriminate against background radiation is essential for any SP experiment. Discrimination against long wavelength background is particularly important, but suitable band pass filters for the wavelength region 10-1000μm are available. The gratings considered here should also be readily available. In order to ensure 'perfect' electrical conductivity, they should be made on a solid metal substrate; aluminium or copper would be suitable for this purpose.

SUMMARY AND DISCUSSION

We have considered some of the theoretical and practical issues associated with the use of coherent SP radiation for the determination of the time profile of $\sim$ 50fs long electron bunches. SP radiation is particularly attractive for this application because it offers wide spectral coverage through the simultaneous use of multiple gratings, with different periodicities. This, in turn, facilitates the bunch profile reconstruction by Kramers-Kronig analysis. Using realistic beam parameters and assuming an experimental arrangement of three gratings with periods of 15, 85 and 500 μm, it was shown that an array of 11 PE detectors is capable of measuring the SP radiation over the range of 10-1000 μm, provided the number of electrons/bunch is $\geq 10^9$. Coherent SP radiation is thus a relatively inexpensive alternative or complementary technique to the transverse deflecting cavities or to Electro-Optic sampling

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