PARTICLE ENERGY DETERMINATION TECHNIQUE BASED ON WAVEGUIDE MODE FREQUENCY MEASUREMENT*

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Abstract

We consider the particle energy measurement method based on detection of the mode frequencies in waveguide loaded with certain material. For this technique, it was found that the mode frequencies should be considerably dependent on the particle energy. A critical issue for this method implementation is the properties of the loading material. With this paper, it is shown that the possible solution found as a system of parallel wires with specific coating deposited of the wire surfaces. The approximate analytical approach for obtaining an effective permittivity of such structure has been developed. It is shown that selection of parameters of the structure allows controlling an effective permittivity characterized by both spatial and frequency dispersions. The structure can be easily fabricated and allows measurement of the particle energy for various predetermined ranges.

INTRODUCTION

Cherenkov radiation is widely used for detection of charged particles and in beam diagnostics [1]. We present here a new method of detection of the charged particles energy based on the measurements of the waveguide mode frequencies [2–7]. For this method implementation, it is important to provide a strong modes frequencies ω_m dependence on Lorentz-factor $\gamma = \left(1-\beta^2\right)^{-1/2}$ of the charged particle.

It was found that the perspective material for the waveguide loading is an anisotropic material with the following tensor of permittivity:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \tag{1}$$

$$\varepsilon_\perp = \varepsilon_c, \qquad \varepsilon_\parallel = \varepsilon_c - \omega_p^2\left(\omega^2 + 2i\omega_d\omega\right)^{-1}, \tag{2}$$

where ω_p is a plasma frequency, ω_d is an attenuation parameter, and ε_c is the arbitrary constant exceeding 1. Strong dependence of the mode frequencies on the particles energy has been demonstrated in our study results published in [3-7]. It should be noticed that the mode frequencies are limited as $\le 20\,\text{GHz}$ providing the mode amplitudes for a relatively short bunch (the length $\le 1\,\text{cm}$) to be considerable to be effectively measured.

With this paper, we consider metamaterials to form a medium with the properties required for particle energy detection. These artificial materials consisting of microelement with relatively small spacing can be treated as some "media" and characterized electromagnetically by effective permittivity and permeability.

The simplest metamaterial is a system consisting of parallel wires. This artificial medium can be characterised by macroscopic permittivity (2) only in the case if the plane wave propagates in the direction orthogonally to the wires. If the wave propagates at another angle an effect of spatial dispersion has to be taken in account [8]. Our calculations showed that this effect is interfering with the energy detection method under consideration. Therefore it is critically important to eliminate or mitigate all influences of spatial dispersion on radiation in a metamaterial of this type. Such problem has been considered in [9], where using the modified structures was proposed. The simplest considered system was a metamaterial formed of the conducting wires with the magnetic coating layers.

However the theory of such structures has not been developed yet. With this paper, we consider wires with ferroelectric coating, and show that not only magnetic coating but as well dielectric one gives taming of spatial dispersion. Furthermore, we show that such structure can be potentially useful for the particle energy detection problem.

EFFECTIVE PERMITTIVITY OF STRUCTURE OF WIRES WITH COATING

Let us consider a periodic system of perfect cylindrical conductors which have a nonconductive cylindrical coating with permittivity ε_1 and permeability μ_1. A radius of conductor is r_0, and a radius of coating is r_d. The surrounding medium is characterized by permittivity ε_2 and permeability μ_2. Let the z-axis is parallel to wires. The periods of system along x- and y-axis are equal to d. It is assumed that the following conditions are fulfilled:

$$r_d << d << \Lambda, \tag{3}$$

where $\Lambda \equiv \min(\lambda, \Delta)$, λ is a typical wavelength, Δ is the distance of variation of the "incident" field (i.e. the field in the case of absence of the structure).

It is clear that the thing wires have essential influence only on the parallel component $\varepsilon_\parallel$ because transversal currents are negligible. Therefore we can consider that the orthogonal component $\varepsilon_\perp$ is determined by permittivity ε_1 and ε_2. The background constant ε_c is determined by

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ε_1 and ε_2 as well. Approximately, we can assume that $\varepsilon_\perp \approx \varepsilon_2 \approx \varepsilon_c$ and estimate this constant with use of the Garnett's formula [13]. It is important for us that these constants can be varied into some range owing to varying $\varepsilon_1, \varepsilon_2, \mu_1, \mu_2$.

The important problem is finding the dependence of $\varepsilon_{||}$ on frequency and wave vector. For this goal, we use new method which is similar to the method of averaged boundary conditions (ABC). The ABC technique was developed in works of V.M. Kontorovich and his followers (see [10] and its references), B.Ya. Moyges [11], and A.V. Tyukhtin [12–14]. These works are devoted to planar periodic structures. Now we use the basic ideas of the ABC method for finding the effective dielectric constant of the 3-D periodic system. This approach can be named "the method of averaged material relations" (by analogy with "the ABC method" for planar structures).

We omit describing the offered method because of limited size of the paper and give only the main result for the structure under consideration. According to it, the parallel component of effective permittivity tensor (1) is written in the form

$$\varepsilon_{||} = \varepsilon_c \left\{ 1 - \frac{\omega_p^2}{\omega^2 - \chi c^2 k_z^2 / (\varepsilon_2 \mu_2)} \right\}, \tag{4}$$

where

$$\omega_p^2 = \frac{2\pi c^2}{d^2 \varepsilon_c \mu_2 \left[\ln\left(\frac{d}{r_d}\right) + \frac{\mu_1}{\mu_2} \ln\left(\frac{r_d}{r_0}\right) - 1.048737 \right]}, \tag{5}$$

$$\chi = 1 + \frac{\left(\varepsilon_2/\varepsilon_1 - \mu_1/\mu_2\right) \ln(r_d/r_0)}{\ln(d/r_d) + \mu_1 \mu_2^{-1} \ln(r_d/r_0) - 1.048737}. \tag{6}$$

In the particular case of wires without coating

$$\chi = 1, \quad \omega_p^2 = \frac{2\pi c^2}{\varepsilon_2 \mu_2 a^2 \left[\ln\left(d\, r_0^{-1}\right) - 1.048737 \right]}. \tag{7}$$

One can see that the role of spatial dispersion is connected with the magnitude of χ. It is important particularly that we can vary strongly this parameter owing to varying magnitudes of $\varepsilon_1/\varepsilon_2$, μ_1/μ_2, and r_d/r_0. Increase in both the coating permittivity ε_1 and the coating permeability μ_1 results in decreasing the role of spatial dispersion (Fig.1). Thus, for taming spatial dispersion, we can use the coating with high permeability or high permittivity. One can see that the use of high permeability μ_1 is more effective for this goal (compare top and bottom pictures in fig.1). However it is difficult to produce a magnetic material without essential conductivity. Therefore use of material with high permittivity is seemed more preferable.

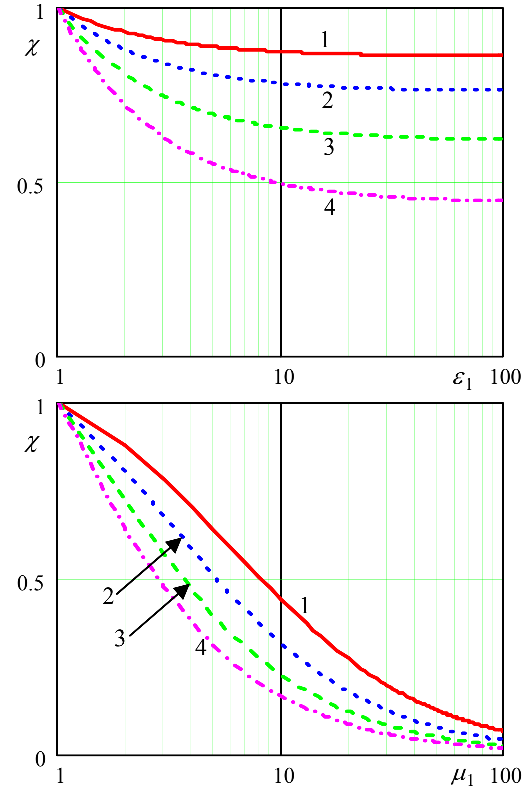

Figure 1: Dependences of the parameter χ on the coating permittivity (top) and the coating permeability (bottom) for the following parameters: $\varepsilon_2 = \mu_2 = 1$, $d = 10\,\text{mm}$, $r_0 = 0.2\,\text{mm}$; $r_d = 0.3\,\text{mm}$ (curve 1), $r_d = 0.4\,\text{mm}$ (2), $r_d = 0.6\,\text{mm}$ (3), $r_d = 1\,\text{mm}$ (4); $\mu_1 = 1$ (top), $\varepsilon_1 = 1$ (bottom).

USE OF STRUCTURE OF PARALLEL WIRES FOR MEASUREMENT OF PARTICLES ENERGY

Let us consider the case when the structure under consideration fills up a circular waveguide, and the main axis of structure (z-axis) coincides with the waveguide axis. It is assumed that $\mu_1 = \mu_2 = 1$. A charged particle bunch moves along the z-axis with a velocity $\vec{V} = c\beta\vec{e}_z$ (agreeably, a Lorentz factor is $\gamma = \left(1-\beta^2\right)^{-1/2}$). The transverse dimension of the bunch is negligible, and longitudinal distribution of the charge is determined by the Gaussian function $\exp\left(-\zeta^2/\left(2\sigma^2\right)\right)$, where $\zeta = z - Vt$ and σ is much less than the typical wavelength.

Using analysis basing on the mode expansion of the wave field behind the bunch (by analogy with [2–7]) one can obtain the following expression for the mode frequencies:

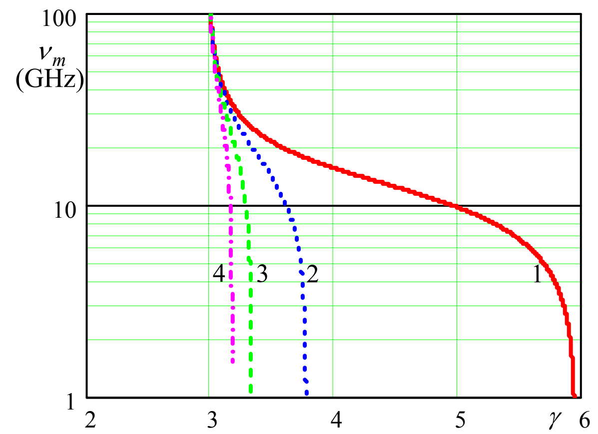

Figure 2: The mode frequencies depending on γ for the medium parameters $\varepsilon_2 = \mu_2 = \mu_1 = 1$, $\varepsilon_c = 1.0027$, $\nu_p = 4.17\,\text{GHz}$, $\chi = 0.887$ corresponding to the wire structure with $d = 10\,\text{mm}$, $r_0 = 0.2\,\text{mm}$, $r_d = 0.3\,\text{mm}$, $\varepsilon_1 = 5$; the mode numbers are indicated in figure.

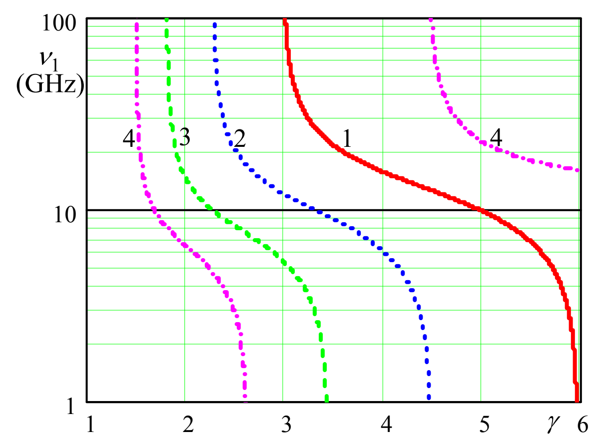

Figure 3: The 1st mode frequency depending on γ for $r_d = 0.3\,\text{mm}$ (curve 1), $r_d = 0.4\,\text{mm}$ (2), $r_d = 0.6\,\text{mm}$ (3), $r_d = 1\,\text{mm}$ (4); other parameters are the same as in Fig.2.

$$\omega_m \equiv 2\pi\nu_m = \beta\sqrt{\frac{c^2\kappa_m^2}{a^2\left(\beta^2\varepsilon_c - 1\right)} + \frac{\varepsilon_c\omega_p^2}{\beta^2\varepsilon_c - \chi}}, \qquad (8)$$

where a is a waveguide radius, κ_m are roots of Bessel function ($J_0(\kappa_m) = 0$).

In the particular case of wires without coating ($\chi = 1$) the modes frequencies are real only for $\beta^2\varepsilon_c > 1$, i.e. Cherenkov radiation is generated only for superlight speed of charge as in an usual medium. The dependence of frequencies on β is strong only for $\beta \approx \varepsilon_c^{-1/2}$ where frequencies are relatively large, and amplitudes for real bunches are small because of the factor $\exp\left(-0.5\,\omega_m^2\sigma^2V^{-2}\right)$ [2–7]. These circumstances show that system of non-coated wires do not have preferences as compared with the ordinary media.

Principally different result takes place in the case of wires with coating. The ordinary modes are generated for $\beta^2\varepsilon_c > 1$ (these modes can be named "normal"). However there are additional ("anomalous") modes under condition $\chi < \beta^2\varepsilon_c < 1$. Frequencies of these modes decrease with increasing the mode number and with increasing γ (Fig.2, 3). The strong dependence $\nu_m(\gamma)$ takes place for relatively low frequencies where the modes amplitudes are not small. Thus, these modes give essential advantages for measurement of particles energy in comparison with modes in waveguide with ordinary material.

Note that some other structures can be perspective for particle energy measurement as well. One of them is a waveguide with a thin cylindrical layer of an ordinary nondispersive dielectric [7]. This method is convenient for low-precision measurement in relatively wide range of particle energy, whereas the system of parallel wires can be used for the high-precision measurement in relatively narrow range of energy.

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