SIMULATIONS FOR MEASUREMENTS OF LONGITUDINAL BUNCH PROFILE USING COHERENT SMITH-PURCELL RADIATION *

Dai Wu#, Wenxin Liu, Chuanxiang Tang
Department of Engineering Physics, Tsinghua University, Beijing, 100084, P.R.China
Key Laboratory of Particle & Radiation Imaging (Tsinghua University), Ministry of Education, Beijing, 100084, P. R. China

Abstract

The coherent Smith-Purcell radiation (CSPR) has been demonstrated as an efficient technique for measuring the longitudinal profile of beam bunches. To measure the ultrashort beam bunches; the simulations for the measurements using CSPR are analyzed with tools of three dimensional particle-in-cell simulations and Kramer-Kronig reconstruction. Different parameters such as RMS length of beam bunch and profiles of grating are studied.

INTRODUCTION

Electron beam bunch longitudinal profile measurement is an active research domain in recent years. In the case of International Linear Collider the bunch length is likely to be a few picoseconds. It is of great interest in the context of X-ray free electron lasers, in which the bunch length is likely to in the 100fs range. In these cases, there is a need of tools for high quality, nondestructive diagnostics for the bunch length and time profile.

A large sum of diagnostic tools for the bunch length is performed, such as streak camera [1], electro-optic sampling (EOS) [2], deflecting cavity [3] and coherent radiative processes, etc. They have their own advantages and disadvantages. Streak camera has fast response and can measure single-shot beam, but it is costly, destructive and of its resolution is only a few picoseconds. EOS can measure ultrashort and single-short beam and has resolutions of sub-picosecond, but its equipment is complicated and costly. Deflecting cavity is mostly widely used, with advantages of high resolution, high response and slice emittance diagnostic capabilities. But because it needs a screen to read the bunch longitudinal profile, it cannot be a nondestructive tool and be used in high average power cases.

With a coherent radiative process, such as transition, diffraction, or synchrotron radiation [4], the measurement of radiated energy can be used to reconstruct the time profile of the bunch, which has wide applicability, high temporal resolution and great simplicity. In recent years, another coherent radiation process, coherent Smith-Purcell Radiation (CSPR) is invited to measure the bunch longitudinal profile. S.E.Krobly from MIT [5] and V. Blackmore [6] with their own research teams successfully used CSPR to measure picosecond and subpicosecond beam bunches, respectively.

In this paper, CSPR is researched as a tool of beam longitudinal profile diagnostic by means of particles-in-cell simulations and Kramers-Kronig reconstruction. The beam bunch parameters are fit for Tsinghua Thomson scattering based X-ray source (TTX) [7].

PHYSICAL MODEL

We consider a single electron (or an electron bunch) moving past a grating of period d with velocity βc and the height x_0 above the grating surface. The diagrammatic sketch is shown in Figure.1. Smith-Purcell Radiation is emitted from the grating surface, with the wavelength relationship [8]:

$$\lambda = \frac{d}{n}\left(\frac{1}{\beta} - \cos\theta\right) \tag{1}$$

in which θ is the angle between the radiation and the radiation and y-z plane, and φ is the angle from the radiation to the x-z plane; n is the order of radiation and λ is wavelength. The radiation therefore disperses according to the observation angle. Shorter wavelengths appear in the forward direction and longer ones the backward direction.

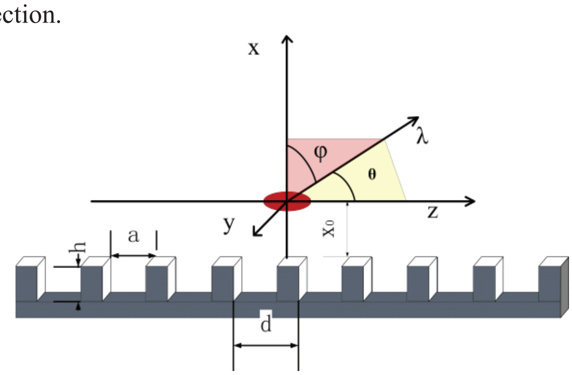

Figure 1: Diagrammatic sketch of CSPR process and the coordinate system.

In the case of a narrow beam bunch with electron number of N, the radiation energy per solid angle is given:

$$\left(\frac{dW}{d\Omega}\right)_N = \left(\frac{dW}{d\Omega}\right)_1 \left(NS_{inc} + N^2 S_{coh}\right) \tag{2}$$

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#Correspondence author: wu-d04@mail.tsinghua.edu.cn

where S_{inc} and S_{coh} are called incoherent factor and coherent factor, respectively. They are related to the bunch profile and the grating profile, etc.

The coherent Smith–Purcell Radiation is generated when the bunch length is less than or comparable to the radiation wavelength. Then the radiation intensity is increase in factor of N^2. In this case, expression (2) can be approximated as:

$$\left(\frac{dW}{d\Omega}\right)_N \approx \left(\frac{dW}{d\Omega}\right)_1 N^2 \left|\int_{-\infty}^{\infty} Te^{-i\omega t}dt\right|^2 \quad (3)$$

Let $F(\omega)=\left|\int_{-\infty}^{\infty} Te^{-i\omega t}dt\right|^2=(\rho(\omega))^2$, which is called longitudinal bunch formation factor.

in expression (3), $\left(\frac{dW}{d\Omega}\right)_1$ is the energy emitted by a single electron and can be calculated numerically which makes formation factor $F(\omega)$ can be get by measuring emitted energy. Then by the tool of Kramers-Kronig (KK) relationship, the bunch longitudinal profile will be reconstructed.

The Fourier transform of bunch profile $T(t)$ can be given as:

$$T(\omega)=\int_{-\infty}^{\infty} Te^{-i\omega t}dt=\rho(\omega)e^{i\psi(\omega)} \quad (4)$$

where amplitude function $\rho(\omega)=\sqrt{F(\omega)}$,assuming $\ln(T(\omega))\neq 0$ and $\lim_{\omega\to\infty}\ln(T(\omega))=0$, phase function $\psi(\omega)$ can be get by KK relationship:

$$\psi(\omega)=-\frac{2\omega}{\pi}\int_0^{\infty}\frac{ln(\rho(\omega))}{x^2-\omega^2}dx \quad (5)$$

The bunch profile will be reconstructed as:

$$T(t)=\frac{1}{\pi}\int_0^{\infty}\rho(\omega)\cos[\psi(\omega)-\omega t]d\omega \quad (6)$$

SIMULATION DESCRIPTIONS

The PIC simulations are carried out with a code of CHIPIC [9]. It is a finite-difference, time-domain code for sufficiently simulating plasma physics process. With the help of 3-dimensional PIC simulation, the characteristics of CSPR power and field distribution as well as the interaction processes of the electron bunch with the grating can be observed.

The simulation geometry is shown in Figure. 2 (a). The cathode consisting of perfect conductor is located at the left, from which an electron beam is emitted. The Observation Plane located at z=0 makes observation angle varying from almost 0 to $\pi/2$. The grating has a period length of 0.8mm, depth of 0.25mm and width of 0.5mm.The beam bunch bottom is 0.3mm higher than the grating top. The operation area is set in vacuum condition and is divided meshes with rectangular cell (δz=δx=0.05mm, δy=0.1mm).

During the simulations, the bunch shape in the longitudinal direction is Gaussian distribution and the total charge is 0.5nC, the length of bunch (RMS) is 0.5ps. The bunch energy is 15MeV. The transverse size is $\sigma_x=\sigma_y=1$ mm and the bunch shape in the transverse direction is uniform. Figureure.2 (b) shows the particle distribution in the x-z plane when 15000 macro particles are used. This bunch profile can be achieved approximately by the TTX source.

According to wavelength relationship (1) and the observation angle, the formation factor after 0.3THz can be calculated better in the simulation, which is shown in Figure 2 (c).

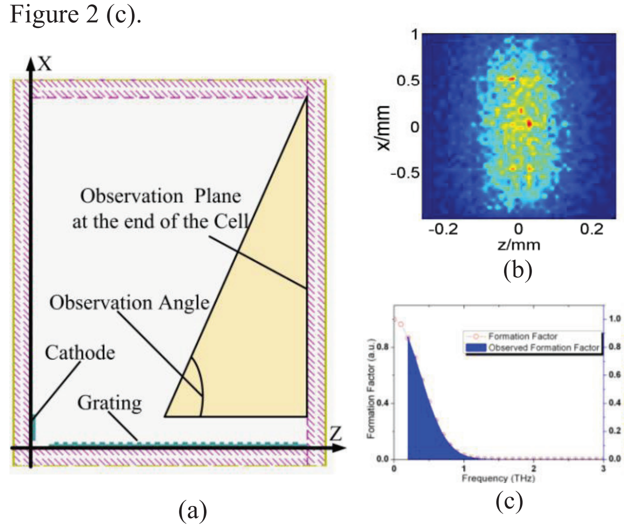

Figure 2: Simulation Geometry (a), particle distribution in the x-z plane used in the simulation (b), and observed formation factor (c) in the simulation.

In order to remove the background radiation's interference, a conductor plane as thick as the grating is used in the simulation to play a role of 'blank' grating.

In practice, the CSPR radiation can be collected by optical parabolic mirror and sent to a Martin-Puplett Interferometer (MPI) device in order to get a power spectrum. This MPI is loaded on the TTX beamline, easy to disassemble, and in good work conditions. The MPI has measured frequency of a THz laser [10]. A further CTR experiment with its help is under performance.

SIMULATION RESULTS

The contour of the magnetic induction in the y direction is given in Figure.3, from which the SPR propagating to the observation plane and be obviously observed.

The frequency form radiation field through the Fourier transform (FFTs) versus different types of grating is shown in Figure.4, from which the advantages of rectangular grating can be found, because its power is a little higher than the other two in this simulation. In the simulation, the total time is 0.3ns. From Figure.4, the total power can be read as about 10^4 Watts, which makes energy of the single bunch Smith-Purcell Radiation is about 10^{-6} Joule.

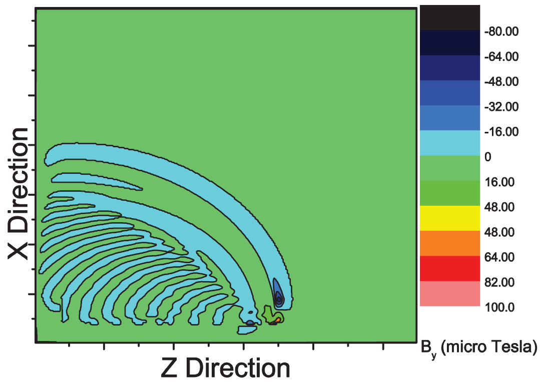

Figure 3: The contour of magnetic induction of y-direction at simulation time t=38.5ps.

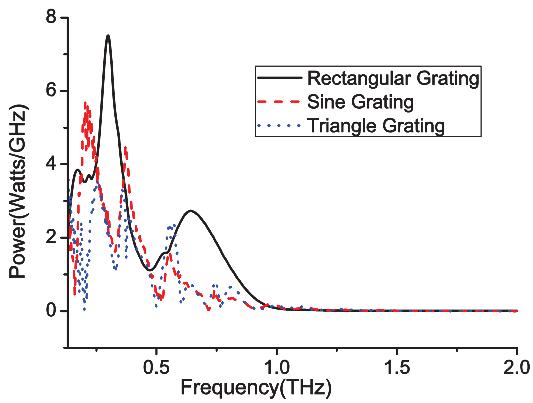

Figure 4: Power of CSPR versus Frequency using different types of grating.

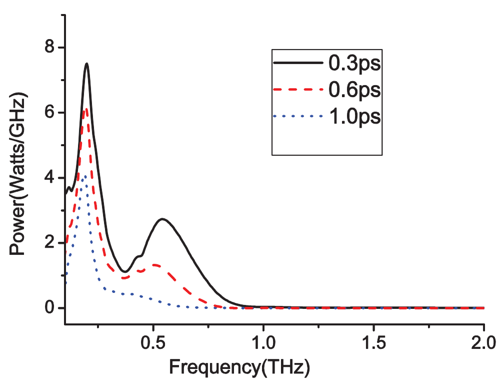

Figure 5: Smith-Purcell Radiation power through the observation plane by different length beam bunches.

By choosing rectangular grating but different beam bunch length, different power spectrum can be simulated, which are shown in Figure.5. Because of the same beam charge 0.5nC is used, the shorter bunch length is, the stronger radiation is generated.

With calculating the expression (3) and single electron SPR energy spectrum, different bunch longitudinal profiles are reconstructed by the tools of KK relationship, which are given in Figure.6. Some negative distortion and bunch length bias exist in the reconstructed profile for three main reasons. Firstly, the observation angle is limited, so as to the formation factor. Secondly, the observation plane is too close to the SPR source, which makes the numerical calculation not exact. Thirdly, KK relationship brings some distortion itself.

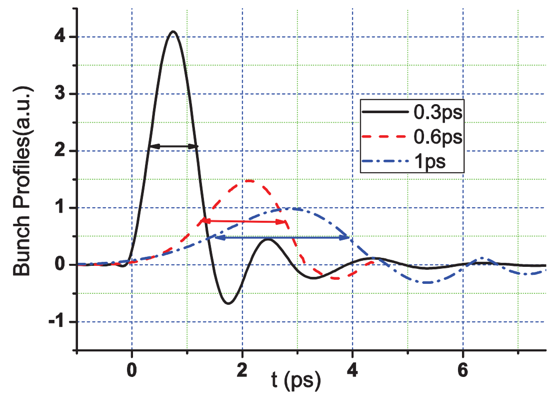

Figure 6: KK reconstruction of these bunches longitudinal profiles.

CONCLUSIONS

In this paper, the CSPR generated from an ultrashort beam bunch is studied as a tool of bunch longitudinal profile diagnosis. With the help of PIC simulation and KK relationship, different bunch profiles are reconstructed. They are in good agreements with the assumptions and the TTX electron beam source. The CSPR could be a new, high quality and nondistructive bunch longitudinal diagnosis tool.

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