UNCERTAINTY MODELLING OF RESPONSE MATRIX

S. Gayadeen, M.T. Heron and G. Rehm, Diamond Light Source, UK

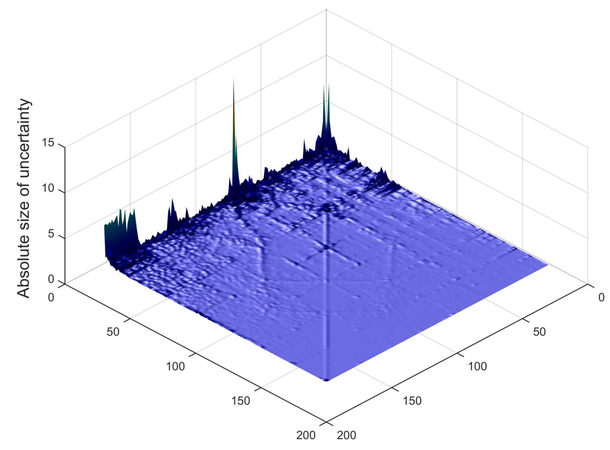
UNCERTAINTY DESCRIPTIONS

- The difference between the ideal and real response matrices of a storage ring is referred to as uncertainty.

  $\Delta_R = R_{\text{real}} - R_{\text{ideal}}$

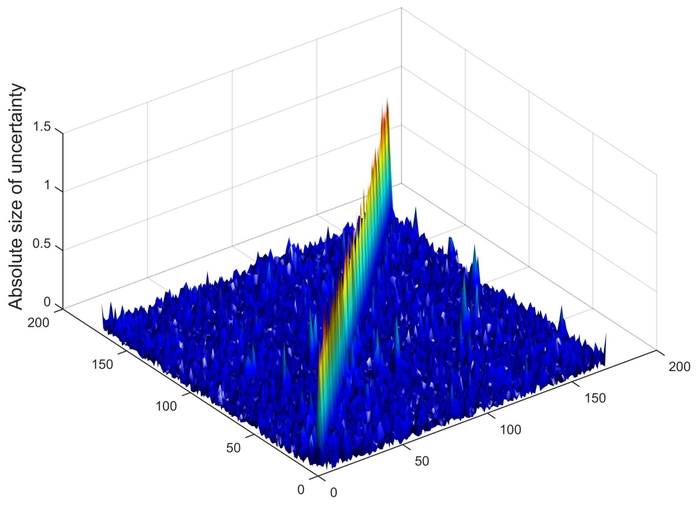
- The uncertainty in response matrix combines BPM scaling errors, corrector scaling errors and tune drift and it is not straightforward to distinguish one source of error from the other.
- Instead it is useful to model uncertainties in process outputs (BPM values) and process inputs (Corrector values).
- Using a Singular Value Decomposition (SVD): $R = \Phi\Sigma\Psi^{\mathrm{T}}$
- Using Fourier analysis : $R = \hat{\Phi}\hat{\Sigma}\hat{\Psi}^{\mathrm{T}}$

Uncertainty in Response Matrix



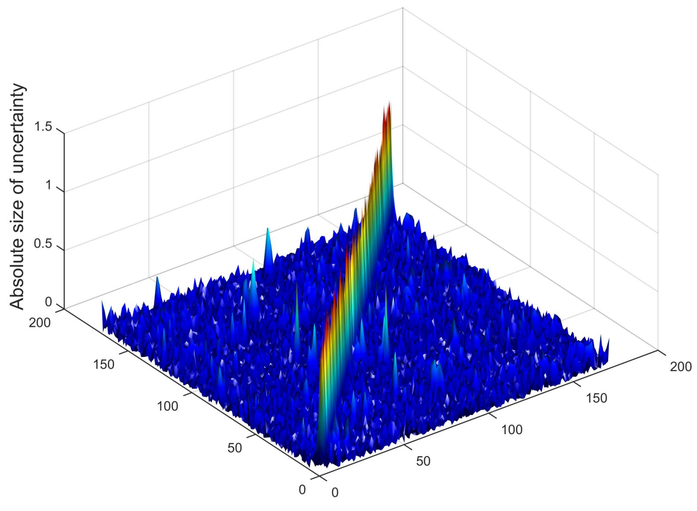
UNCERTAINTY DESCRIPTIONS USING SINGULAR VALUE DECOMPOSITION

Uncertainty in BPMs using SVD



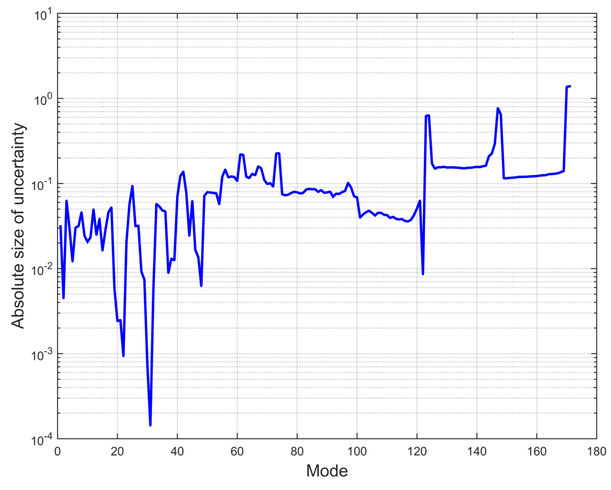
- Uncertainty in BPMs is expressed by comparing the Φ matrices of the ideal and real response matrices.
- The shape of the uncertainty has a strong diagonal component but there are also non-diagonal effects meaning that **there is correlation to other BPMs in these modes.**

Uncertainty in Correctors using SVD



- Uncertainty in the correctors is expressed by comparing the Ψ matrices of the ideal and real response matrices.
- The shape of the uncertainty has a strong diagonal component but there are also non-diagonal effects meaning that **there is correlation to other correctors in these modes.**

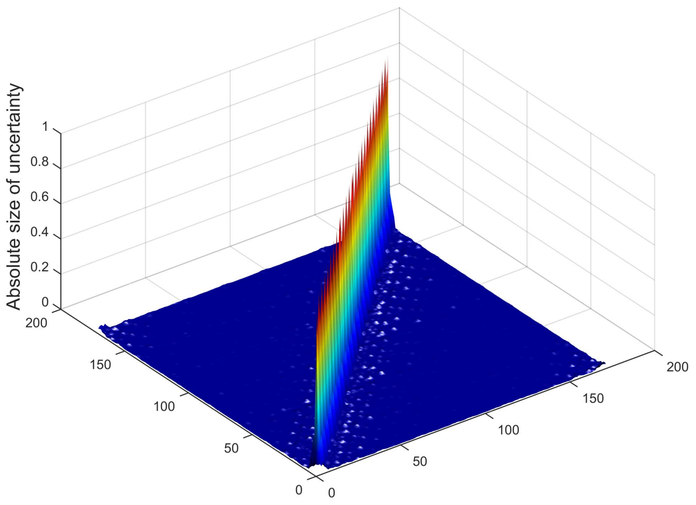
Uncertainty in Singular Values



- Uncertainty in the singular values can be expressed by comparing the Σ matrices of the ideal and real response matrices which is diagonal.
- The size of the uncertainty in the higher order modes is greater than the error in low order modes meaning that **the high order modes are more vulnerable to stability problems.**

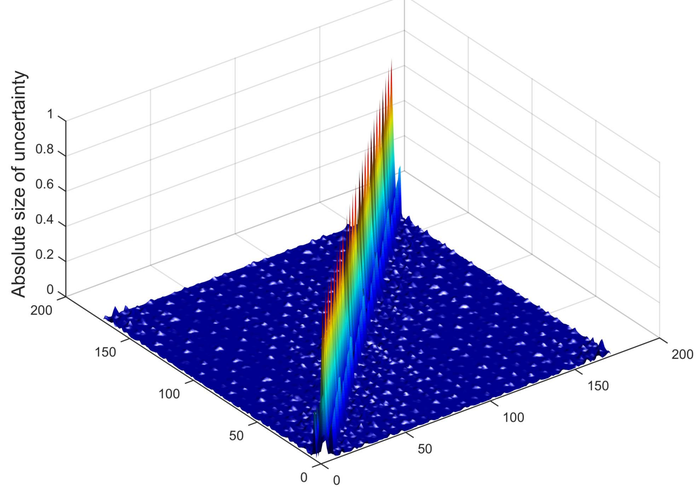
UNCERTAINTY DESCRIPTIONS USING HARMONIC DECOMPOSITION

Uncertainty in BPMs using Harmonic Decomposition



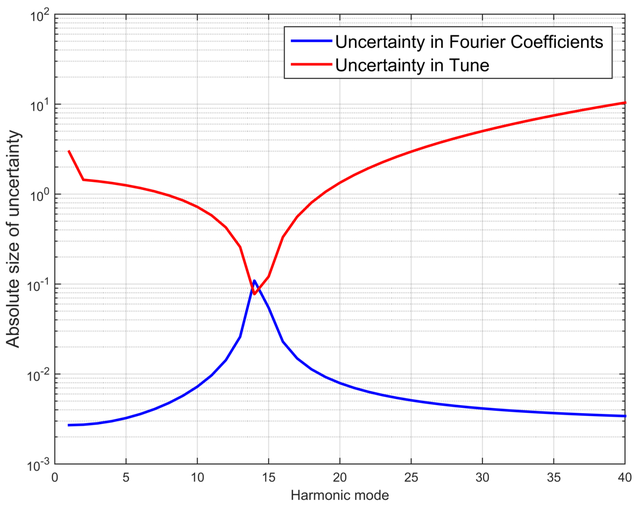
- Uncertainty in the process outputs can be expressed by comparing the $\hat{\Phi}$ matrices of the ideal and real response matrices.
- The matrix $\hat{\Phi}$ depends solely on the beta function and phase advance at BPM locations. **Coupling between the beta function and phase advance within a cell results in off-diagonal elements being non-zero**.

Uncertainty in Correctors using Harmonic Decomposition



- Uncertainty in the process inputs can be expressed by comparing the $\hat{\Psi}$ matrices of the ideal and real response matrices.
- The matrix $\hat{\Psi}$ depends solely on the beta function and phase advance at corrector locations. **Coupling between the beta function and phase advance between cells results in off-diagonal elements being non-zero**.

Uncertainty in Fourier Coefficients



- The uncertainty in the Fourier coefficients is diagonally structured and the largest error is seen at the modes associated with the tune i.e. the 13th mode for tune = 13.36. **The harmonic modes closest to the tune are more vulnerable to stability problems.**
- The Fourier coefficients are solely determined by the tune, therefore **the size of uncertainty in the tune is determined for each harmonic mode**.

For more information please visit www.diamond.ac.uk or contact Sandira Gayadeen at sandira.gayadeen@diamond.ac.uk