HEAD-TAIL BUNCH DYNAMICS WITH SPACE CHARGE

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Head-Tail Instability

SIS100 of FAIR: the major concern at the injection plato

modified by Space Charge ($\Delta Q_{sc}$ up to 0.25)

$$\Delta Q_{sc}(\tau) = \frac{\lambda(\tau)r_p R^2}{\gamma^3 \beta^2 Q_0 a^2}$$

not considered by classical (Sacherer) theories

here: single-bunch instability, below the coupling threshold
A non-realistic bunch, but a very useful model
M. Blaskiewicz, PRSTAB 1, 044201 (1998)

square-well (barrier) potential $\Rightarrow$ constant line density $\Rightarrow$ constant $\Delta Q_{sc}$

longitudinal distribution $f(p) \propto \delta(p - p_0) + \delta(p + p_0)$

Assumptions: rigid flows, $Q_0 \gg |\Delta Q|$, arbitrary space charge

without wake: $\Delta Q = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{\frac{\Delta Q_{sc}^2}{4} + k^2 Q_s^2}$ “+” for $k \geq 0$

space charge parameter $q = \frac{\Delta Q_{sc}}{Q_s}$
Recent analytic works:
A.Burov, PRSTAB 12, 044202 (2009); A.Burov, PRSTAB 12, 109901(E) (2009)
V.Balbekov, PRSTAB 12, 124402 (2009)

OUR APPROACH: PARTICLE TRACKING SIMULATIONS

PIC codes: PATRIC

HEADTAIL
G.Rumolo and F. Zimmermann, PRSTAB 5, 121002 (2002)

this work: “semi-frozen” electric field, homogeneous transverse profile,
the space charge implementation verified using the airbag theory
Gaussian bunch: Gaussian line density, Gaussian momentum distr.
Simulations for the Gaussian bunch
Spectrum example for $q=20$,
red dashed line: the airbag theory

Tune shifts are well predicted by the air-bag theory, especially for stronger SC
Bunch form seems to be not very important!

$$q = \frac{\Delta Q_{sc}}{Q_s}$$
Landau damping in bunches exclusively due to the effect of space charge discussed in [Burov 2009], [Balbekov 2009]

\[ \Delta Q_{sc}(\tau) = \frac{\lambda(\tau)r_pR^2}{\gamma^3\beta^2Q_0\alpha^2} \]

Note: in a coasting beam space charge CAN NOT produce Landau damping of its own!

We start a simulation with an initial perturbation of a head-tail mode.
LANDAU DAMPING DUE TO SC

Summary of Landau damping simulations: damping decrement for a Gaussian bunch
Eigenfunctions for a Gaussian bunch: extracted in simulations using Landau damping (at $q=6$)

theory [Burov 2009]:

$$\dddot{y} + \nu \exp(-q^2/2)\dot{y} = 0$$

airbag head-tail eigenfunctions [Blaskiewicz 1998]

$$\bar{x}_k(\tau) = A_0 \exp(-i\xi Q_0 \tau/\eta) \cos(k\pi \tau/\tau_b)$$

$$\tau_b = \frac{4\sigma_z}{R}$$

eigenfunctions of a Gaussian bunch are very close to the airbag modes!
Landau damping can be seen in the bunch spectrum

$q=5$

$q=20$

the modes $k=2, k=3$ suppressed

here not
LANDAU DAMPING DUE TO SC

**a physical interpretation**

upper boundary of the incoherent effective spectrum: small \( \Delta Q_{sc} \), large synchrotron amplitudes

\[
\Delta Q_{\text{max}} \approx -0.23Q_s q + kQ_s
\]

red: \( k=1 \); blue: \( k=2 \)
a physical interpretation

upper boundary of the incoherent spectrum:
small $\Delta Q_{\text{sc}}$,
large synchrotron amplitudes

$$\Delta Q_{\text{max}} \approx -0.23Q_s q + kQ_s$$
Landau damping:
resonant particles with small $\Delta Q_{sc}$ have large synchrotron amplitudes,
energy transfer happens in the bunch tails

this leads to the local emittance increase:

horizontal rms beam size along the bunch,
a simulation for $k=2$, $q=3$
airbag bunch: analytical results for unstable head-tail modes [Blaskiewicz 1998]

\[ W(\tau) = W_0 \exp(-\alpha \tau) \]
\[ \alpha \tau_b \gg 1 \]

for \( k = 0 \)
(not affected by space charge)

\[ \Delta Q = \Delta Q_0 (\alpha / \zeta + i) \]
\[ \Delta Q_0 = -\frac{\kappa \zeta}{\alpha^2} W_0 \]

for \( k > 0 \)

\[ \Delta Q = -\Delta Q_{sc} + \frac{\Delta Q_0 (\alpha / \zeta + i) + \Delta Q_{sc} \pm \sqrt{[\Delta Q_0 (\alpha / \zeta + i) + \Delta Q_{sc}]^2 + 4k^2 Q_s^2 [1 - (\Delta Q_0 \pi / 2 \zeta Q_s \tau_b)^2]}}{2 [1 - (\Delta Q_0 \pi / 2 \zeta Q_s \tau_b)^2]} \]
Simulations for a Gaussian bunch:
growth rates of the most unstable head-tail mode

\[ \chi = \xi Q_0 \tau_b / \eta = 4.2 \]

Landau Damping!

wake function of the thick resistive wall:

\[
W_{rw}(z) = -\frac{cL_{rw}}{b^3} \left( \frac{\beta}{\pi} \right)^{3/2} \frac{Z_0}{\sqrt{z \sigma_{rw}}} \]

\[ q = 8 \]

\[ q = 16 \]
HEAD-TAIL INSTABILITY WITH SC

simulations: the growth rates saturate at strong space charge

analytic theory for the airbag bunch [Blaskiewicz 1998]

calculations in [Burov 2009]:
- treating a wake as a perturbation
  ➔ diagonal element of the wake operator

\[
\Delta Q = \frac{\kappa}{N_{\text{ion}} \lambda_0 R} \int_0^{z_b} dz \int_{s}^{s + z_b} ds \ W(s - z) d_k(s) d_k^*(z)
\]

\[
d_k(s) = \lambda(s) \bar{x}_k(s)
\]

provides the growth rate at strong space charge

comparisons with the simulation results show a good agreement!
Landau damping in bunches exclusively due to space charge observed in particle tracking simulations. The damping strength depends on $q = \Delta Q_{sc}/Q_s$ and on the mode index $k$. The energy transfer happens in the bunch tails.

Gaussian bunch: the transverse functions and frequencies are very similar to that predicted by the airbag theory [Blaskiewicz 1998]

Simulations of the head-tail instability with space charge: Landau damping can stabilize the bunch. The instability growth rates saturate at strong space charge (compared to [Blaskiewicz 1998] and [Burov 2009]).

Examples:
- CERN PS (E. Métral et al, PAC07) $q \sim 150$: far above Landau damping, growth rates saturated
- FAIR SIS100 $q \sim 20$: Landau damping might contribute to the stability