Transverse Schottky spectra and beam transfer functions of coasting ion beams with space charge

Stefan Paret, Vladimir Kornilov, Oliver Boine-Frankenheim, GSI, Germany,
Thomas Weiland, Technische Universität Darmstadt, Germany
Outline

• FAIR and SIS-18
• Schottky diagnostics and beam transfer functions
  – Effect of linear space charge
• Measurement of space-charge effects
• Simulation of space-charge effects
• Summary
FAIR: experiments with high quality and high intensity beams

**SIS-18** becomes booster
- Increase of beam intensity
- Arise of collective effects
  - Degradation of beam quality and particle losses
- Low energy
  - Strong space charge

P. Spiller, MOIC01
Low intensity Schottky spectrum

- Based on statistical fluctuations of local beam current and current dipole moment
- Non-destructive measurement of
  - Revolution frequency $f_0$
  - Fractional tune $Q_f$
  - Momentum spread
- Features
  - Longitudinal bands peaking at $f_0 m$
  - Side bands $P_0(f)$ centered around $f_0 (m \pm Q_f)$
  - Width of sidebands $\sigma_{m \pm}$
Schottky detection

Requires

- Pick-up
- Sum amplifier for longitudinal spectrum
- Difference amplifier for transverse spectrum
- Spectrum analyzer

![Diagram showing beam, spectrum analyzer, pick-up, and amplitude graph with f/ MHz and P/W axes.]
Transverse beam transfer functions (BTFs)

- BTF $r_0(f)$ defined as ratio of beam response to excitation
- Requires
  - Network analyzer
  - Exciter (kicker)
  - Pick-up
  - Difference amplifier
- Alternative to Schottky diagnosis
- Stability analysis
Impedance and space charge

- Impact of transverse dipolar impedances
  - Coherent tune shift $\Delta Q_{coh}$
  - Coherent dipolar instability with growth rate $\tau$—if not Landau damped
  - Impedance parameters
    \[
    \Delta U_{coh} = \frac{\Delta Q_{coh} f_0}{\sigma_m^\pm} \quad \text{and} \quad \Delta V = \frac{1}{\tau \sigma_m^\pm}
    \]

- (Direct) space charge
  - Non-linear self-field, very difficult to model
    $\rightarrow$ tune spread
  - Linearized self-field (of K-V beam)
    $\rightarrow$ incoherent tune shift
    \[
    \Delta Q_{sc} \propto \frac{N}{\epsilon}
    \]
Diagnostics with collective effects

High intensity BTF [1] and Schottky band [2]

\[
r(f) = \frac{r_0(f_{sc})}{1 - (\Delta U_{coh} + i\Delta V - \Delta U_{sc})r_0(f_{sc})}
\]

\[
P(f) = \frac{P_0(f_{sc})}{|1 - (\Delta U_{coh} + i\Delta V - \Delta U_{sc})r_0(f_{sc})|^2}
\]

with \(\Delta U_{sc} = \frac{\Delta Q_{sc}f_0}{\sigma^\pm_m}\) and \(f_{sc} = f + \Delta U_{sc}\sigma^\pm_m\)


Diagnostics with collective effects

High intensity BTF [1] and Schottky band [2]

\[
r(f) = \frac{r_0(f_{sc})}{1 - (\Delta U_{coh} + i\Delta V - \Delta U_{sc})r_0(f_{sc})}
\]

\[
P(f) = \frac{P_0(f_{sc})}{\left|1 - (\Delta U_{coh} + i\Delta V - \Delta U_{sc})r_0(f_{sc})\right|^2}
\]

with \(\Delta U_{sc} = \frac{\Delta Q_{sc}f_0}{\sigma_m^\pm}\) and \(f_{sc} = f \mp \Delta U_{sc}\sigma_{m}^\pm\)


Diagnostics with collective effects

High intensity BTF [1] and Schottky band [2]

\[ r(f) = \frac{r_0(f_{sc})}{1 - (\Delta U_{coh} + i\Delta V - \Delta U_{sc})r_0(f_{sc})} \]

\[ P(f) = \frac{P_0(f_{sc})}{|1 - (\Delta U_{coh} + i\Delta V - \Delta U_{sc})r_0(f_{sc})|^2} \]

with \( \Delta U_{sc} = \frac{\Delta Q_{sc} f_0}{\sigma_m^\pm} \) and \( f_{sc} = f \pm \Delta U_{sc} \sigma_m^\pm \)


Experimental setup

- Energy 11.4 MeV/nucleon
- Detection of
  - Ion number $N$
    varied from $2.5 \times 10^8$ to $1.1 \times 10^{10}$ Ar$^{18+}$ ions
  - Longitudinal Schottky Spectra
    → Gaussian momentum distribution
  - Beam profiles
    with ionization profile monitor → emittance
- $\Delta U_{coh}, \Delta V \ll \Delta U_{sc}$ → only $\Delta U_{sc}$ taken into account
Measured Schottky bands

- Fit of
  \[ P(f) = \frac{P_0(f_{sc})}{1 + \Delta U_{sc} r_0(f_{sc})^2} \]

- Good agreement at low, medium and maximal intensity
Measured BTFs

• Noise suppression via time gating

• Fit of $r(f)$
  – Good agreement at low intensity
  – Deviations at high intensity

Amplitude [a. u.]
Phase [rad]
Measured stability diagrams

- Stability diagram with space charge
  \[
  \frac{1}{r(f)} = \frac{1}{r_0(f_{sc})} + \Delta U_{sc}
  \]

- Shifted as expected

- Approximately shaped as expected

- Disturbed by noise at high intensity
Measured space-charge parameter

- Estimation with beam parameters $\rightarrow \Delta U_{est}$
- Deformation of signal $\rightarrow \Delta U_{shape}$
- Position of signal ($f_{sc}$) $\rightarrow \Delta U_{shift}$
- Consistency $\rightarrow \Delta U_{shift} - \Delta U_{shape} = 0$

- $\Delta U_{sc}$ grows linearly with $N$
- Measured $\Delta U_{sc}$ larger than estimation
- Larger $\Delta U_{sc}$ for BTF
Possible error sources

**Beam parameters**

- Uncertainty of beta function at profile monitor
- Degradation of detector components

**BTFs**

Beam of high intensity close to coherent instability

- Nonlinear response to excitation?
- Perturbation by resonance?
PIC simulations

• Random macro particle distribution in phase space
  Fluctuation of dipole moment → transverse Schottky spectrum
• Self-consistent field computation in 2D
• Options:
  – Excitation with noise for BTF
  – Impedance kicks
• Transverse profiles: K-V beam or Gaussian
• Maximal $\Delta U_{sc} = 2$
Schottky simulations

Results for beam with Gaussian transverse profile

\[ \Delta U_{sc} = 0 \]
\[ \Delta U_{sc} = 1 \]
\[ \Delta U_{sc} = 2 \]

- \( \Delta U_{sc} \) fitted to data
- Excellent agreement with data and expected \( \Delta U_{sc} \)
- Similar results for K-V und Gaussian profiles
BTF simulations

Results for beam with Gaussian transverse profile

- $\Delta U_{sc} = 0$
- $\Delta U_{sc} = 1$
- $\Delta U_{sc} = 2$

- $\Delta U_{sc}$ fitted to data
- Excellent agreement with data and expected $\Delta U_{sc}$
- Similar results for K-V und Gaussian profiles
Simulated stability diagrams

- Good agreement with model
- More noise at high intensity
Simulation with impedance

Variation of $\Delta U_{coh}$ and $\Delta U_{sc}$ for direct comparison

Shift and deformation agree with model
Summary

**Analytic linear space-charge model**

- Different from dipolar impedance

**Experiment**

- Measurement of transverse Schottky spectra and BTFs
- Verification of model despite deviations in some parts
- Direct measurement of $Q$, $\Delta Q_{sc}$ und $\Delta U_{sc}$

**Simulation**

- Transverse Schottky spectra and BTFs with space charge and imaginary impedances
- Excellent agreement with model
Thank you for your attention
Measured $\Delta Q$