**BEAM-BEAM SIMULATIONS FOR FUTURE ELECTRON-ION COLLIDER eRHIC**

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**Abstract**

The future electron-ion collider eRHIC - under design at BNL - will collide the electron beam accelerated in energy recovery linacs with protons or ions circulating in the RHIC storage ring. The beam-beam effects in the linac-ring configuration have a number of unique features. For the in-depth studies of the beam-beam effects and the resulting luminosity limitations, we developed a dedicated simulation code. We studied the effects of the mismatch, the disruption and the pinching on the electron beam. Relevant dynamics of the proton beam, including the kink instability in combination with incoherent beam-beam effects, was also explored in detail. In this paper we describe the main features of our simulation code and present the most important simulations results.

**INTRODUCTION**

Several designs of electron-ion colliders are under development in the world [1]. The design of electron-ion collider eRHIC at BNL adds an electron accelerator, based on energy recovery linacs (ERLs), to the existing heavy ion accelerator complex RHIC [2]. The eRHIC design uses a so-called linac-ring collision scheme. The electron beam, accelerated in the ERL, passes a collision point just once, while the proton (or ion) beam circulates in a ring and passes the collision point on every turn. There may be several collision points in the collider, although in this paper we show the simulation results for the case of one electron-proton collision.

Since the electron beam goes through the collision point(s) only on one pass, the allowed strength of the beam-beam force acting on the electron beam can be much large than for electrons circulating in a storage ring. Thus a typical beam-beam limit for electrons in circular colliders can be surmounted. The resulting eRHIC luminosity in the linac-ring scheme is considerably larger than that in the ring-ring scheme. Present eRHIC design aims at the luminosity of e-p collisions exceeding $10^{34}\text{cm}^{-2}\text{s}^{-1}$.

The studies of the beam-beam interactions in the linac-ring collision scheme is very important eRHIC R&D item. Since there has been no collider based on the linac-ring collision layout, there is no any operational experience with the linac-ring beam-beam interactions and the related machine performance limits. Thus all features of beam collisions in the linac-ring scheme have to be thoroughly studied during the machine design. This would allow to determine the maximum achievable luminosity and to identify and address possible problems originating from the beam-beam interactions. One should note that the linac-ring collider scheme have been considered in previous years for accelerator designs, for example, as possible design for B-factory. Hence, the specific features of the linac-ring beam-beam interactions had been also studied [3,4]. For eRHIC, the following features of the beam-beam interactions have to be considered and investigated:

- The electron beam disruption. The level of the disruption should be acceptable for the electron beam transport and deceleration in the ERL.
- The electron beam pinch, the related enhancements of the luminosity and the beam-beam effect on the proton beam.
- The kink instability of the proton beam.
- The effect of fluctuating electron beam parameters (intensity, transverse emittance) on the proton beam.

A comprehensive study of the list above require a full-blown simulations of the beam-beam effects including the nonlinearity of beam-beam force, the variation of beta-function throughout the collision region, synchrotron oscillations of the proton beam, chromaticity and amplitude-dependence of proton betatron tunes. A code EPIC was created [5] to carry out the detailed and time-efficient studies of the beam-beam effects in eRHIC. The following section provides description of the EPIC simulation code. In later sections we present some results of the beam-beam simulations and discuss the influence of those results on the collider design.

**THE BEAM-BEAM INTERACTION MODEL AND SIMULATION CODE**

The EPIC code takes into account two considerable asymmetries in the eRHIC collision scheme. One is the asymmetry of the strength of the beam-beam force acting on the electrons and the protons. Both in terms of the beam-beam parameters ($\xi_p=0.015$, $\xi_e=2.2$), and in the terms of the disruption parameters ($D_p=0.007$, $D_e=27$) the beam-beam effect on the electron beam is much stronger compared with that on the hadron beam. Because of the strong beam-beam effect the electron beam gets disrupted during the pass through the collision region. In contrary to that, the beam-beam effect on the protons is moderate, and the effect of the interactions becomes important on the scale of thousands and million turns. The strong asymmetry of the beam-beam effects is used in the EPIC simulation code to separate the study of one pass effect of the electron beam disruption and multi-turn effect of the beam-beam interaction on the proton (ion) beam.
Another asymmetry is the difference in the bunch lengths of electron and proton beams ($\sigma_{le} = 2\text{mm, } \sigma_{lp} > 5\text{cm}$). Hence, the short electron beam can be considered as an infinitely thin slice, when simulating the beam-beam effect on the protons. The EPIC code uses two presentations of the beam-beam force. In one presentation, the force is calculated assuming the Gaussian transverse distribution. For eRHIC where both beams are round at the collision point, the force expression is simplified to the radial force:

$$F_r = \frac{ne^2}{\pi \varepsilon_0 r} \left[ 1 - \exp\left(\frac{-r^2}{2\sigma^2}\right) \right]$$  \hspace{1cm} (1)

where $n(s)$ is the longitudinal charge density and $\sigma$ is the transverse beam size. In the EPIC code both full form of the beam-beam force (1) as well as linearized presentation of this force can be used.

In second presentation, the force is calculated for the round beam using the Gauss law. This force presentation can be used for calculating the force produced by the infinitely thin electron beam:

$$F_r ds = \frac{N_e(r)e^2}{\pi \varepsilon_0 r}$$  \hspace{1cm} (2)

Here $N_e(r)$ is the number of electrons within the radius $r$.

The approach realized in the EPIC code for the simulation of the beam-beam interaction consists of the interconnected and consecutive applications of two “strong-weak” simulations, one for the propagation of the electron macro-particles through the field of the proton beam and another for the propagation of the proton macro-particles through the field of the electron beam.

**Simulation of the Effects on Electron Beam**

Since the distribution of the proton beam is only weekly affected by the beam-beam force in one pass, for the study of the electron disruption the proton beam can be considered undisturbed. Therefore, the strong-weak scheme can be applied in the simulations. The proton beam is divided in longitudinal direction in multiple slices. Each slice is treated as an infinitely short bunch with a transverse Gaussian distribution generating the electric field according to the equation (1). Typically the use of 20 (or more) slices is adequate for providing consistent results. The longitudinal Gaussian distribution of the proton beam is considered Gaussian with tail cut-off typically selected at $4\sigma_p$. The variation of the transverse rms beam size of the proton beam due to the variation of the proton beta-function throughout the collision region is taken into the account in the slicing procedure.

The electron beam is represented by macro-particles that experience consecutive kicks from the interactions with the proton slices. The macro-particles can be generated with a desirable initial transverse distribution (Gaussian or Beer-Can distributions has been usually used in the simulations). Symplectic integrator up to $3^{rd}$ order is used to propagate the electron macro-particle through the sequence of the proton beam slices. Following the modification of the electron beam distribution during the interaction process, the electron beam parameters, such as the transverse emittance, the beam size, transverse distribution moments, optical functions (beta and alpha functions) can be calculated by post-processing the macro-particle data. Usually, at least 50 thousand macro-particles have been used for the electron distribution. In order to determine the electron beam and optics parameters throughout the collision region, $3^{rd}$ order spline function is applied to the corresponding data points calculated at the proton slice locations. The collision luminosity, modified by the electron beam pinching, is also calculated.

**Simulation of the Effects on Proton Beam**

In order to study the multi-turn effect of the beam-beam interaction on the proton beam, the proton beam is presented by a collection of macro-particles. Initially the macro-particles are distributed in all three dimensions according to the input beam size parameters and then they are propagated through the field of the electron beam. As was already mentioned, due to the short length of the electron beam, the electron beam can be considered as an infinitely thin slice.

On each turn the interaction of the proton beam with the electrons is considered in two steps. On first step the effect on the electron beam is evaluated using the approach described in the subsection “Simulation of the Effects on Electron Beam”. As the result, the data for continuous evolution of the electron transverse beam size and the electron slice transverse position throughout the collision region are obtained. Thus, on the second step, the beam-beam force can be calculated either using (1) with the electron beam size or using (2) with $N_e(r)$ dependence. The force calculation is done at the proper longitudinal coordinate of the interaction of the electron beam slice and a proton macro-particle. The electron slice transverse position offset is also taken into the account in the calculation.

Following the beam-beam interaction, the proton beam is transported through the one turn of the accelerator ring, using one turn transformation matrix. The one turn transformation includes the effect of the chromaticity and amplitude dependent betatron tune. It also executes the synchrotron oscillations in the longitudinal plane.

Since at every turn the protons encounter a new electron beam, coming from the linac, the initial parameters of the electron beam can be varied from one turn to another.

On the basis of the obtained simulation data for the proton macro-particles the multi-turn evolution of the proton beam emittances and the transverse orbit offsets can be obtained using the data post-processing.

Further details on the EPIC code can be found in [5].
ELECTRON BEAM DISRUPTION

Here we present selected simulations results for the effects on the electron beam [6]. The electron beam, after passing a collision point, has to be decelerated in ERLs and transported in recirculation lines. The distortion of the electron beam distribution has to be kept at an acceptable level in order to allow the beam transport. The deformation of the electron distribution by the nonlinear beam-beam force can be considered in two ways: as a mismatch of the beam distribution relative to the one defined by the lattice function in the absence of the beam-beam force; and the increased particle population at large betatron amplitudes, including a halo formation. Both effects can be minimized by a proper selection of the design interaction region optics (β* and s*). Besides that the optimal selection of the design optics should avoid an excessive pinching the electron beam size, since it can be harmful for the proton beam.

Figure 1 demonstrates an example of the evolution of transverse electron beam parameters throughout a collision area from EPIC simulations. The effective emittance describes a transverse emittance, which is based on the phase ellipse defined by the design lattice functions. The geometric emittance is calculated from the beam distribution, using a statistical definition of the emittance and optics functions.

Figure 1. The evolution of electron transverse beam size and emittance values throughout the collision area. Shown is a case for electrons with 5 GeV energy. The disruption parameter D=27.

Figure 2 shows that a proper choice of the design β* and s* values allows to minimize unwanted modifications of the e-beam distribution by the beam-beam interactions. The electron beam disruption studies have been used to finalize our choices for the IR optics and the electron beam emittance. The study also set the requirements on the aperture of recirculating magnets and ERL components. Used electron beam has to be decelerated and the particles pushed to high amplitudes by the beam-beam interactions can cause a beam loss on the ERL apertures. Figure 3 shows the example of such evaluation.

Figure 2. The transverse phase space distribution of electrons after passing the collision area for two IR optics and transverse emittances. Top plot: β* = 5 cm, s* = 0 cm, εrms = 2.1 nm. Bottom plot: β* = 2.5 cm, s* = 3 cm, εrms = 4.2 nm.

Figure 3. The beam loss power at given aperture of recirculation pass magnets at several beam energies during the beam deceleration after the collisions. The results for two kinds of the initial transverse distribution are shown. Actual distribution is expected to be somewhere between these two cases.

KINK INSTABILITY

Several beam-beam effects can cause deterioration of the proton beam. The kink instability is one of these effects, which we had studied. It is a strong head-tail instability, where the interaction between head and tail of a proton bunch is provided by the electron beam. Although there have been several theoretical papers on
the subject [7,8], they all are based on approximation. The complete process with a nonlinear beam-beam force, a realistic longitudinal distribution and a variation of beta-function in the collision area can be obtained only from the simulations.

![Figure 4](image)

Figure 4. The snapshot of the beam centroid position along the collision area as the kink instability develops.

Our initial simulations verified basic features of the instability such as the dependences on the proton beam intensity and the bunch length, the corresponding mode frequency shifts, and the pattern of the instability development above the threshold (Figure 4).

![Figure 5](image)

Figure 5. The increase of beam emittance due to the kink instability for different proton chromaticities.

Our simulation studies showed strong dependence of the instability threshold on the proton chromaticity. It was found that at least seven units of the chromaticity is needed to prevent the instability at design beam parameters (Figure 5).

The large value of the chromaticity and the corresponding large betatron tune spread may limit an available betatron tune space and may complicate the machine operation. For instance, the proton beam chromaticity in RHIC is typically kept at 2 units.

Hence, we considered a dedicated feedback to damp the kink instability as an alternative remedy [9]. In the feedback scheme, the transverse position of an electron bunch is measured by a dedicated position monitor after it passed the IR. The kicks calculated on the basis of these measurements is applied to the incoming electron bunches by a kicker is located before the collision point.

Using the EPIC code we demonstrated that such feedback is feasible.

**THE EFFECT OF ELECTRON PINCH ON PROTON BEAM**

In addition to the coherent instability, the transverse emittance of the proton bunch can be blown up in an incoherent way by the beam-beam force. In the EPIC code we can study the incoherent effects separately from the kink instability by suppressing the relative dipole motion of the electron bunch and the proton slices.

![Figure 6](image)

Figure 6: The proton transverse emittance growth for various of design parameters.

Focusing force of proton beam deviates considerably the electron beam size throughout the collision area from its design pattern (Figure 1). Although the effect of electron beam pinching can lead to the luminosity enhancement, in the same time it enhances the beam-beam force acting on the protons. It can become unacceptably large. In addition, the variation of the electron beam size both due to pinching pattern and the design variation of beta-function (“hour-glass”) lead to a modulation of the beam-beam parameter during the synchrotron oscillation of protons. It can cause the appearance of synchro-betatron resonances. Since the deviation of the electron’s distribution from the Gaussian is significant, the field induced by the electron beam is calculated by the method (2). The simulations showed that the luminosity correlates well with the average electron beam size throughout the collision area. The electron beam size pinching pattern depends on the electron beam optics. The simulation studies were used to find optimal parameter choices, which provide high luminosity without deterioration of the proton emittance. The Figure 6 shows an example of EPIC simulation results for the incoherent emittance growth. We note that Blue and Red curves
correspond to the cases with same resulting luminosity, however the emittance growth is greatly different.

THE ELECTRON BEAM NOISE

Fluctuations of the electron beam parameters, such as the transverse emittance, the bunch intensity and the transverse position offset, could affect the proton beam emittance. The fluctuations of the position offset lead to the random dipole kicks at beam-beam interactions. The fluctuations of the electron transverse emittance and bunch intensity led to the fluctuations of the beam-beam force focusing. The EPIC simulations confirm well analytical formulas for the case of white noise spectrum of the fluctuations [10,11]. Simulations with more realistic frequency spectra characterizing laser stability errors, magnet errors and earth’s movement are planned. The simulations should help to establish tolerances on those errors.

Figure 7: Proton rms beam size evolution at the presence of the electron bunch intensity noise and the comparison with the theoretical anticipation. Each data point represents the average of 1000 turns.

CONCLUSIONS

The EPIC code was written for simulating the beam-beam effects for the linac-ring collision scheme in a time efficient way. The simulations, using EPIC code, have been crucial for finalizing the interaction region lattice and beam parameters. A proper choice of electron IR optics functions and the transverse emittance allows us to minimize both the electron beam disruption and proton beam emittance growth. The kink instability, observed in the simulations, can be cured either by the (sufficiently large) chromaticity or by a dedicate feedback. The study of electron beam parameter fluctuations will help to establish tolerances on errors of various kinds (laser stability, magnet errors, …).

In near future we plan to study the interplay between the beam-beam interactions and proton beam space charge. This capability will be added to the EPIC code. Also, the combination of beam-beam effects from multiple collision points and the interplay between the beam-beam interactions and coherent electron cooling would be considered.

REFERENCES