OVERVIEW OF BEAM OPTICS IN PROJECT-X SC CW LINAC∗
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Abstract
Project-X is a proposed multi-MW proton facility at Fermilab. Based on a new superconducting H⁻ linear accelerator, it would provide the foundation for a flexible long term intensity frontier physics research program. Two machine configurations have been developed. The first one involved a single 8 GeV, pulsed linac (9 mA peak, 1 ms @ 5 Hz pulses) followed by accumulation and acceleration to 60-120 GeV in the existing Main Injector synchrotron. The second -and currently favored one- replaces the single pulsed linac by a 3 GeV (10 mA peak, 1 mA average), continuous wave linac followed, up to 8 GeV, by either a rapid cycling synchrotron or a second (pulsed) linac. We present here an overview of beam optics for the 3 GeV CW linac. Alignment, field amplitude and phase tolerances are also addressed.

INTRODUCTION
The US elementary particle physics community strategic plan for the coming decade emphasizes research on three frontiers: the energy, intensity and cosmic frontiers. As the sole US site for accelerator based particle physics research, Fermilab’s strategy features the development of a high intensity, multi-MW proton source. This new facility, dubbed Project-X, is based on a superconducting H⁻ linear accelerator. Project-X will provide the flexibility to support diverse intensity-frontier physics experiments, including a world leading program in neutrino physics. Ultimately, it would serve a basis for a future neutrino factory and/or muon collider. Specifically, the objectives of Project-X are:

• provide 2 MW of beam power at a beam energy of 60 to 120 GeV for long base line neutrino oscillation experiments
• provide > 1 MW of high intensity low energy protons for rare decay experiments operating simultaneously with the neutrino program.
• provide a path toward a muon source for a future Neutrino factory and/or Muon collider: 4 MW of beam power at 5-15 GeV.

Historical Background
The genesis of Project-X is the Fermilab Proton Driver (PD), a concept developed at the beginning of the decade [1, 2]. The PD was an 8 GeV pulsed superconducting H⁻ linac used to inject and accumulate beam into the existing Main Injector synchrotron. To capitalize on the ILC (then TESLA) technology, the PD front-end frequency (325 MHz) was selected to be a submultiple of the 1.3 GHz ILC frequency. The PD featured a single four-fold jump in frequency to 1.3 GHz around 400 MeV with the bulk of the acceleration (from 2.4 to 8 GeV) subsequently handled with unmodified ILC cavities. An innovative scheme involving fast ferrite phase shifters for independent cavity phase and amplitude control was also introduced.

Project-X
At an early stage, Project-X retained many ingredients of the PD concept, most notably the 8 GeV pulsed linac. A subsequent series of reviews, studies and workshops led to the conclusion that the 8 GeV pulsed linac lacked the flexibility necessary to support both the near and long term Fermilab physics programs. An optimal energy for planned rare-decay experiments was deemed around 3 GeV. Perhaps more importantly, different experiments required simultaneous operation with vastly different beam timing structures. These considerations led to the current concept for Project-X (technically referred to as IC-2.2), and shown schematically in Fig. 1. It consists of a 3 GeV continuous-wave (CW), 1 mA average, 10 mA peak linac, followed by rf separators to dispatch portions of the beam to different experiments. The chief advantage of CW operation is that it allows for arbitrarily complex beam pulse structures to be accelerated. The beam structure is imposed at low energy, before acceleration in the linac using a fast broadband chopper and can modified more or less at will without altering the main linac operation. An added benefit of CW operation is that it is inherently more stable than pulsed operation. To reach 8 GeV, two options are being consid-
ered: a pulsed 8 GeV linac using ILC-style cavities, or a rapid cycling synchrotron. 8 GeV beam would ultimately be accumulated in the Main Injector/Recycler complex and accelerated up to 120 GeV. Possible siting for Project-X, assuming the pulsed 8 GeV linac option, is shown in Fig. 2. In this paper we focus on optics design of the 2.5 - 3.0 GeV linac. While a variety of scenarios have been explored we discuss here two representative iterations of the linac optics. The first one is considered our baseline design and involves 325, 650 MHz and 1.3 GHz (ILC) cavities for acceleration. The second one eliminates the 1.3GHz cavities and uses 650 MHz cavities exclusively to reach 3 GeV. We shall refer to it here as the he650 lattice.

LINAC DESIGN PROCEDURE

In designing linac optics, the overall objective is to minimize the potential for emittance growth and losses by producing a stable beam envelope that is as smooth and regular as possible. Theoretical advances, numerical simulation results and operational experience have produced a number of rules and guidelines that can be followed to attain this objective. In proton or ion linacs, accelerated particles experience a strong transverse rf defocusing kick resulting in marked influence of longitudinal on transverse dynamics, at zero current. In contrast, the transverse dynamics affects the longitudinal dynamics more weakly, mostly through space charge. For this reason, the design starts in the longitudinal phase plane. Because the particle velocity increases with increasing energy, synchronism with the accelerating cavity fields must be preserved as much as possible to achieve efficient acceleration. This ideally implies many independently phased cavities, each with a geometry optimized for a particular value of β. It is important to provide within a relatively short period, rapid acceleration so as to reduce space charge forces early. This is accomplished by putting resonators and focusing elements in a common cryostat. This approach, used consistently in the entire linac also minimizes the number of costly and complex cold/warm transitions and maximizes the real-estate gradient. At superconducting temperatures, in particular at low energy, short solenoids are an attractive choice as transverse focusing elements: they provide, in a compact package, pure radial focusing to match the radial defocusing due to space charge and rf fields. Quadrupole doublets (or triplets) may also be used, but generally they consume more precious longitudinal space and introduce locally non-radial transverse focusing. In the lattices discussed here, solenoids provide transverse focusing in the periods involving SSRs; doublets are introduced later in the low-energy (β = 0.6) 650 MHz section when more focusing force is needed and radial defocusing effects have become comparatively weaker.

Envelope Stability

In a periodically focusing system, single particle trajectories are known to be stable as long as the phase advance per period at zero current σ₀ satisfies the condition σ₀ < 180°. In the presence of space charge, it can be shown [3] that envelope instabilities can arise when σ > 90°, which is somewhat more restrictive. Envelope instabilities are automatically avoided by conservatively setting the focusing strengths in each plane so as achieve a value below 90°.

Parametric Resonances

While the phase advances are typically the same in both transverse planes, longitudinal and transverse oscillations are parametrically coupled through the dependence of the transverse rf defocusing strength on the phase. A simplified analytical model show that such resonances occur when σ₀.4 = 4σ₀. The longitudinal phase advance should be chosen so as to avoid the strongest one, n = 1, which is usually the only one of significance.

Table 1: Cavity Types for Project X

<table>
<thead>
<tr>
<th>Frequency</th>
<th>βopt</th>
<th>Name</th>
<th>Energy Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MHz</td>
<td></td>
<td></td>
<td>MeV</td>
<td></td>
</tr>
<tr>
<td>325</td>
<td>0.11</td>
<td>SSR-0</td>
<td>2.5-10</td>
<td>single spoke</td>
</tr>
<tr>
<td>325</td>
<td>0.21</td>
<td>SSR-1</td>
<td>10-32</td>
<td>single spoke</td>
</tr>
<tr>
<td>325</td>
<td>0.4</td>
<td>SSR-2</td>
<td>32-160</td>
<td>single spoke</td>
</tr>
<tr>
<td>650</td>
<td>0.6</td>
<td>beta06</td>
<td>160-520</td>
<td>5-cell elliptical</td>
</tr>
<tr>
<td>650</td>
<td>0.9</td>
<td>beta09</td>
<td>520-3000</td>
<td>5-cell elliptical</td>
</tr>
<tr>
<td>1300</td>
<td>1.0</td>
<td>ILC</td>
<td>2000-3000</td>
<td>9-cell elliptical</td>
</tr>
</tbody>
</table>

Figure 2: Project-X possible siting, shown here assuming the high energy pulsed linac option.
Focusing Field Strengths

At constant field strength, the transverse focusing strength decreases inversely with $\sqrt{\beta\gamma}$, as do the transverse emittances. The result is that the envelope amplitude remains approximately uniform provided the focusing field strength is held constant from period to period. Additional insight may be gained by inspection of the envelope equation

$$x'' + k^2(s)x = 0$$

where $k^2(z)$ is the optical focusing strength as a function of the longitudinal position $z$, $\epsilon$ is the emittance, and $<xF_{sc}>$ is the first moment of the (possibly non-linear) space charge force $F_{sc}$. In an idealized uniform focusing channel with acceleration, $k^2(z)$ depends on $z$ via the energy only, equation 1 admits an uniform amplitude solution $x'' = x = 0$ provided the ratio $k^2 <xF_{sc}>$ is independent of energy. Clearly, this is true to a reasonably good approximation when the field in the focusing elements is held constant and the space charge force is a perturbation.

These observations lead to the following prescription: for each linac section, the field strengths of the focusing elements are set to be nominally constant from period to period. The field strength value is chosen so that at the beginning of each section, the transverse phase advance per period is slightly below 90°. Because of the reduction in optical focusing strength caused by acceleration, the phase advance per cell decreases gradually until it reaches a minimum of approximately 20 to 30°, at which point a new section is started with new (different) period and the phases advances $\sigma_{\perp}$ are reset to 90°. Note that phase advance per period at the end of a section should not be allowed to go much below 20° as this would result in a significant increase in sensitivity to misalignments and/or field errors. Parametric resonances with the the longitudinal oscillations will be avoided by keeping $\sigma_{0\parallel}$ in the range 0.6 – 0.8$\sigma_{0\perp}$.

Apertures, Bunch sizes

Transversely, avoiding losses involves keeping the ratio between the physical aperture and the beam size large, typically at least a factor of 5, possibly 10 or more. Longitudinally, the rf synchronous phase sets that limit and the objective is then to ensure that the rms bunch length, measured in rf degrees, remains comfortably below $\phi_s$, the synchronous acceleration phase. Longitudinally, a bucket width to bunch length ratio of 5 is difficult to achieve, especially in a low energy front end. It would probably not be wise to allow this ratio to ever go much below 3.

Transitions

To provide a smooth transition between the regular periodic envelopes of two regular sections, focusing elements strengths on both sides need to be adjusted. Although matching involves a minimum of two constraints per plane, the quality of the match is considerably improved by using more variables. Experience shows that a quality match between sections with different periods is critical to minimize losses. Additional care is needed when a jump in rf frequency is involved.

Even with good matching between sections, the beam envelope will typically exhibit residual irregularities. Small irregularities may also be present within regular periods due to the discrete nature of the acceleration. A well-known result of WKB theory applied to the equation

$$x'' + k^2(z)x = 0$$

is that the equation admits solutions with a smooth, slow varying envelope amplitude provided that $k'' << k'/k$. Not surprisingly, it turns out that the rate of change of the phase advance per unit length $\sigma$, (the smooth approximation version of the wavenumber $k$ in 2 above) provides a sensitive measure of residual envelope irregularities. Conversely, these irregularities can be reduced by minimizing the magnitude of a finite-difference version of the second order derivative of $k$ over many periods. The smoothing process typically involves iterating on the strengths of all focusing elements.

Frequency Jump

A significant difference between longitudinal and transverse dynamics is the fact that longitudinal focusing is inherently more non-linear than its transverse counterpart. This is especially noticeable in the linac front-end, when the bunch is longer and more susceptible to experience the curvature of the rf field. Across a transition where a frequency jump occurs, matching is optimized for a specific nominal longitudinal emittance. However, an increase in emittance (or beam current) with respect to the design value result in a corresponding increase in bunch length. Different nonlinearities on each side of the transition result in mismatch and subsequent emittance growth [6]. To make frequency jump transition robust with respect to this phenomenon, one approach (implemented in the code GenLinwin) is to optimize the acceleration profile so as to keep the bunch length to acceptance ratio constant.

Emittance Transfer and Equipartioning

The beam rest frame rms kinetic energy fluctuations in each phase plane define the beam “temperatures”. In the presence of random energy exchanges between planes, the temperatures will tend to equalize, that is, the emittance in a given plane will grow at the expense of the others. Such random energy transfers necessary for thermalization are favored by space charge fields. Changes in emittance may also be triggered when coupling resonances are excited by space charge. In that case, the emittances in each plane do not equalize, but rather may simply undergo an abrupt “exchange”. To mitigate both effects and preserve emittances along the linac, one approach is to start with an equipartitioned beam, that is, an input beam with equal beam tem
peratures. The equipartitioning condition is [3]:
\[
\frac{T_\perp}{T_\parallel} = \frac{k_x \epsilon_{nx}}{k_z \epsilon_{nz}} \tag{3}
\]
where \(k_x, \epsilon_{nx}\) and \(k_z, \epsilon_{nz}\) are the wavenumber and normalized emittance in the transverse and longitudinal planes respectively. For the Project-X linac lattice studies, we assume \(\epsilon_{nx} = \epsilon_{ny} = 0.25\) mm-mrad and \(\epsilon_{nz} = 0.5\) mm-mrad. Since \(\frac{k_x}{k_z} \approx \frac{\epsilon_x}{\epsilon_z} \approx 0.7\) to avoid parametric resonances, the equipartition condition is only roughly satisfied, which should be adequate given the fact that the linac is operating conformably away from the space charge dominated regime.

Cryo Segmentation

As already alluded to, the resonators and the transverse focusing elements are housed in long cryostats. The linac is divided into distinct sections, each with specific regular period topologies. Each section is comprised of an integer number of of cryostats and each cryostat encompasses multiple periods. The space required for cryostats inter-connection tends to break the lattice periodicity and the end periods within each cryostat should be designed to account for this and mitigate the disruption. Since the cryostats must by necessity be treated as whole units, the optimization procedure must take this constraint into account. So far our experience is that the tools available do not deal with this issue in fully satisfactory way.

CODES

The optimization, matching and smoothing procedures outlined in the preceding paragraphs need to be performed with the assistance of computer programs. Various codes are available, but after exploring a few alternatives, we settled on the suite of codes developed by CEA/Saclay[4]: GenLinWin for longitudinal dynamics optimization, TraceWin for matching and smoothing both in transverse and longitudinal planes. Of note is the flexibility of TraceWin in enabling the user to impose constraints and its ability to transparently switch between a quick and efficient moment-based model and more accurate particle tracking (handled by the PARTRAN module). The codes TRACK[5] and ASTRA[7] have also been used extensively. These codes are primarily particle-tracking codes with capabilities similar to PARTRAN i.e. they track particles through detailed field maps and provide a space-charge solver. Both codes are well-suited for runs in batch mode on a large scale computer grid. To that extent, they have been used primarily for statistical error studies. We also use all codes to run cross-checks.

RESULTS AND DISCUSSION

The cavity types and period topologies for the 3 GeV linac are summarized in Tables 2 and 2. The beam envelopes for the baseline and he650 lattices are shown in Fig. 4. In the baseline lattice, focusing in the 1300 MHz section is provided by FODO-style cells. Longitudinal matching at 2 GeV present some difficulty because of the ineffectiveness of the the rf to affect the bunch length as \(\beta\) approaches 1. Accordingly, a small amount of longitudinal emittance growth (on the order of 10%) is observed after the transition while no meaningful growth occurs in the transverse plane. For the he650 lattice, no growth is observed in any plane. The corresponding emittance plots are shown in Fig. 6. The phase advance per period is shown in Fig. 3. Note the adiabatic variation from slightly below 90° per cell at the beginning of a section down to 30°/cell at the end of a section in transverse phase advance and the fact that the longitudinal phase advance/cell remains around 0.8 times less than than its transverse counterpart everywhere. Finally, plots of the cavity voltage for both lattice variants are shown in Fig. 5. As can be seen, introducing ILC 1.3 GHz cavities at 2 GeV results in some inefficiency, which is in retrospect not surprising given the number of cells (9) in these cavities. The he650 lattice represents a saving of about 30 cavities. The cavity count is summarized in Table 3.

Table 2: Period Topology Used in Different Sections. R: resonator, S: solenoid, D: doublet, QF: focusing quad QD: defocusing quad, R2: R-R, etc.

<table>
<thead>
<tr>
<th>Lattice</th>
<th>SSR0/1</th>
<th>SSR2</th>
<th>BETA06</th>
<th>BETA09</th>
<th>ILC</th>
</tr>
</thead>
<tbody>
<tr>
<td>baseline</td>
<td>S-R</td>
<td>S-R2</td>
<td>R-D-R</td>
<td>R-D-R4</td>
<td>R-D-R4-QF-R3-QD-R4</td>
</tr>
<tr>
<td>he650</td>
<td>S-R</td>
<td>S-R2</td>
<td>R-D-R</td>
<td>R-D-R</td>
<td>R2-D-R4</td>
</tr>
</tbody>
</table>

Table 3: Cavity Counts

<table>
<thead>
<tr>
<th>Lattice</th>
<th>SSR0</th>
<th>SSR1</th>
<th>SSR2</th>
<th>BETA06</th>
<th>BETA09</th>
<th>ILC</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>baseline</td>
<td>26</td>
<td>18</td>
<td>44</td>
<td>42</td>
<td>96</td>
<td>72</td>
<td>298</td>
</tr>
<tr>
<td>he650</td>
<td>26</td>
<td>18</td>
<td>44</td>
<td>36</td>
<td>144</td>
<td>0</td>
<td>268</td>
</tr>
</tbody>
</table>

Figure 3: Beam phase advances/period. Top: baseline lattice, bottom: he650 lattice.
Error Studies

To get an initial assessment of the lattice sensitivity to various errors, studies been performed on the baseline lattice using the code TRACK v39 running on the FermiGrid. For each type of error, the studies involved 400 runs with different random seeds. For these initial studies, no correction was assumed. The results are summarized in Table 4. Detailed analysis of the run results shows that losses occur only when the beam centroid is allowed to wander more than 10 mm off the machine reference axis. A subsequent study, assuming a beam position monitor and a corrector located near each transverse focusing element, random misalignments of 1 mm for all elements, rf jitter $\phi, V$ of 0.5$\degree$, 0.5% in the front end and 1.0$\degree$, 1.0% in the high energy part resulted in no loss (100 seeds, 1.0 $\times 10^6$ particles/seed). While these results are still preliminary, they suggest that tolerances on phase and amplitude jitter may turn out to be the most challenging ones. A realistic static error correction strategy remains to be devised and analyzed.

Table 4: Summary of Error Studies Performed on the Baseline Lattice Using the Code TRACK

<table>
<thead>
<tr>
<th>Error Type</th>
<th>Limit</th>
<th>Lossy Runs / 400</th>
</tr>
</thead>
<tbody>
<tr>
<td>solenoid ($\delta x, \delta y$)</td>
<td>300 $\mu$m</td>
<td>3</td>
</tr>
<tr>
<td>solenoid (pitch angle)</td>
<td>2 mrad</td>
<td>2</td>
</tr>
<tr>
<td>quad ($\delta x, \delta y$)</td>
<td>300 $\mu$m</td>
<td>3</td>
</tr>
<tr>
<td>quad (pitch angle)</td>
<td>300 $\mu$m</td>
<td>0</td>
</tr>
<tr>
<td>cavity ($\delta x, \delta y$)</td>
<td>&gt;10 mrad</td>
<td>0</td>
</tr>
<tr>
<td>cavity (pitch angle)</td>
<td>&gt;10 mrad</td>
<td>6</td>
</tr>
<tr>
<td>RF phase jitter</td>
<td>1$\degree$</td>
<td>20</td>
</tr>
<tr>
<td>RF field jitter</td>
<td>1%</td>
<td>3</td>
</tr>
<tr>
<td>RF field+phase jitter</td>
<td>1$\degree$ + 1%</td>
<td>56</td>
</tr>
</tbody>
</table>

CONCLUSIONS

Project-X has evolved significantly in the last year. The concept of a 3 GeV linac operating in CW mode is now well-established and we have developed optics meeting basic requirements. Much work remains to be done to finalize the design: (1) optimize the cryo-segmentation, (2) modify the optics to accommodate warm regions for instrumentation and diagnostics (3) develop a static error correction strategy (4) perform more exhaustive statistical error studies (5) understand the implications of possible issues with the reproducibility of cavity performance and how this could be mitigated.

REFERENCES