Abstract
The choice of undulator design and minimum magnet gap is crucial in the definition of every short wavelength FEL and is ultimately a cost driver for that project. The magnet gap selection is a compromise between wanting to minimise harmful wakefield effects whilst at the same time generating high magnetic fields with short periods. The NLS project has tried to take a holistic approach in the definition of the undulators. This has been carried out by first assessing the impact of resistive wall wakefields in general on the FEL performance and then selecting the maximum level of wakefield which has a just tolerable impact on the FEL. This wakefield is then translated into equivalent circular and elliptical vessel geometries. Suitable vessel thickness and mechanical tolerances are then added to define the undulator magnet gap for the case of a circular vessel (APPLE-3 and Delta undulators) and an elliptical vessel (APPLE-2 and crossed-planar undulators). Finally, the four types of undulator have been modelled and their parameters compared. This paper summarises this global, self-consistent, approach to undulator definition and reports on the result for the NLS.

INTRODUCTION
The NLS is a UK-based 4th generation light source project with a Science Case [1] that demands high repetition rate, ultrashort, high brightness, high coherence X-rays and a range of other light sources tightly synchronised to these X-rays spanning the THz to VUV range. To realise this goal a unique facility has been designed combining high repetition rate seeded soft X-ray FELs and advanced laser sources.

The initial three FELs will cover the range from 50 eV to 1 keV in the fundamental, with overlapping tuning ranges as follows; 50 to 300 eV, 250 to 850 eV, and 430 to 1000 eV. Harmonics will further extend the output to 5 keV. All of the FELs will provide variable polarisation.

This paper describes the approach taken in order to select the optimum undulator designs for the project taking account of as many relevant aspects as possible, such as wakefield effects on the FEL output, alternative undulator designs, and vacuum chamber geometries.

DESIGN PHILOSOPHY
Often an early decision for a light source project is the type of undulator that will be used and the minimum magnet gap for that undulator. The reason for this early choice is that it is required to help define the electron energy for the facility in conjunction with the wavelength range specified by the user requirements.

For the NLS project we have also made an initial selection of the undulator type and gap in order to make rapid progress with the design of the whole facility but have deliberately reassessed this selection as soon as start-to-end modelling was able to provide bunch profiles at the entrance to the FELs. This second iteration in the undulator parameters will enable the NLS overall design to be optimised before the project invests too much effort in the initial solution and therefore becomes ‘locked in’ to that choice.

The start-to-end bunch profiles from the initial NLS design have been used in a thorough assessment of the wakefield effects as a function of vacuum vessel aperture and shape on the FEL performance. It is important to consider different vessel shapes since some undulator designs require circular vessels and others can be used with elliptical vessels.

This assessment determines the vessel dimensions that are tolerable in terms of FEL output degradation. These ideal values need to be combined with practical vessel wall thicknesses and engineering tolerances in order to determine the minimum magnet gap for the different types of undulator.

Once a minimum magnet gap for each type of undulator is known it is possible to make genuine comparisons between the alternative designs and so gain a clear understanding of the optimum design for the project. The selection of the optimum undulator is likely to lead to a change in the required electron energy. Of course, a significant energy change implies that a further design iteration for the facility is required including reassessment of the wakefield effects.

For the NLS design the initial assumption made for the undulators was that they would be of the APPLE-2 type and that they would operate with a minimum magnet gap of 8 mm and the vessels would have an internal gap of 6 mm [1]. Consideration of the FEL energy range coverage required then led to the NLS electron energy being set to 2.25 GeV and allowed the accelerator design to be progressed [2, 3]. A sketch of the three FELs is given in Figure 1. Note that the FELs are seeded by an HHG source and that they make use of harmonic up-conversion to cover the required photon energy range.
RESISTIVE WALL WAKEFIELDS

A start-to-end model was generated from the photocathode to the exit of the three FELs. The electron bunch longitudinal profile at the entrance to the FELs has been used during resistive wall wakefield calculations to assess the impact of the possible undulator vessels on the electron bunch properties and so on the FEL output performance. Assessments have been made of both aluminium and copper vessels and these have shown there to be little difference between the two materials and so aluminium has been the main focus of this study as it is an easier material to work with in practice for the fabrication of vacuum vessels. The AC conductivity model has been used throughout these studies.

For consideration of elliptical vessels the aspect ratio between the two axes is important. Calculations have been carried out for a variety of ratios $a:b$ (as defined by Figure 2) and these have indicated that, in general, the change in the wake as $a$ is increased beyond $a \sim 3b$ is small.

Some example wakes for the NLS bunch, which has a peak current of $\sim 1.2$ kA, are given in Figure 3 for both circular and elliptical vessels for two different apertures. For the bunch parameters of the NLS the difference between an elliptical vessel and a circular vessel appear to be relatively small.

Our investigations have shown that the relative difference in wake between elliptical and circular vessels is strongly correlated with the overall bunch length, or more precisely, the frequency content of the bunch. A very short bunch will excite very high frequencies and this favours elliptical vessels (i.e. elliptical vessels then have weaker wakefields for the same vertical aperture). It is for this reason that single electron wake functions show that elliptical vessels appear to have a significant advantage. However, as longer bunches are considered the apparent advantage is less clear and this is the case for the NLS. Note that the NLS generates, in absolute terms, a very short bunch with a FWHM of only $\sim 150$ fs.

These findings can be understood by considering the longitudinal impedance for the AC conductivity model, given in Figure 4 for copper using the method described in [4]. When all frequencies are considered the elliptical impedance has a lower impact since it has in general a lower value but at intermediate frequencies the shift in the resonant frequency with chamber aspect ratio can lead to the elliptical case being poorer than the circular one. In general, in comparison with a circular vessel, only an elliptical vessel with $a \sim 1.2b$ is always advantageous at all frequencies and therefore for all bunches.
FEL PERFORMANCE

Time dependent modelling of FEL-3 (see Figure 1), the most demanding of the three NLS FELs in terms of photon energy and overall undulator length, have been carried out using GENESIS 1.3 [5]. The effect of the resistive wall wakefields has been calculated by examining the FEL power output at 1000 eV at saturation as a function of vessel shape and aperture. Only the radiator sections have the wakefield included in the model since the modulator sections are much shorter and less demanding in terms of minimum gap requirement.

Figure 5 shows how the power level in FEL-3 changes for different vessel geometries as a function of vertical aperture. Figure 6 summarises how the peak power level changes for the two cases considered.

![Figure 5: The power output from FEL-3 including the effect of resistive wall wakefields for two different vessel geometries, as a function of different vertical half apertures.](image)

![Figure 6: Peak power output from FEL-3, as a percentage of the output with no wake included in the model, as a function of vertical half aperture for circular and elliptical ($a = 3b$) vessels.](image)

The results show that there is negligible difference between elliptical and circular vacuum vessels on the FEL output for NLS. The results also show that even with a very conservative inner vessel half-aperture of 5 mm the effect on the FEL output of the resistive wall wakefield is significant. If the peak power is instead expressed as a percentage of what is practically realisable (i.e. relative to the 5 mm case) then this would suggest that 3 mm would give 87% and 4 mm would give 94% of the potential peak power output. Assuming (somewhat arbitrarily) that a peak power loss of ~10% is acceptable, it is therefore proposed that the full internal vertical aperture for the NLS undulators should be 7 mm, independent of the actual vessel geometry.

VACUUM VESSEL ANALYSIS

Following the analysis of the vacuum vessel inner dimensions an assessment has been made of the wall thickness required to support the vessel under atmospheric load. For the elliptical case it is assumed that the vessel will be elliptical on the inside and rectangular on the outside so the thinnest point of the vessel walls is directly above and below the electron beam. Finite element modelling has shown that a minimum wall thickness of only 0.25 mm is required at this thinnest point to support the vacuum forces. This thickness is independent of whether the vessel is manufactured from copper or aluminium. Such a narrow walled vessel may suffer from vacuum porosity problems and so careful experimental testing would need to be carried out to finally confirm such a wall thickness. Similar studies have suggested that for a circular vessel the wall thickness would only need to be 0.1 mm to withstand the atmospheric load. Again, vacuum porosity would be an issue for aluminium or copper vessels but a stainless steel vessel coated with either copper or aluminium should be acceptable.

For the elliptical case, with an internal aperture of 7 mm, the addition of a suitable wall thickness and inclusion of allowances for mechanical tolerances, straightness, and alignment suggests that the minimum magnet gap should be 8.1 mm and for the circular case the minimum magnet diameter should be 7.6 mm.

UNDULATOR PERFORMANCE

A number of undulator solutions are possible for the NLS FEL radiator sections. The requirement that the output radiation must have variable polarisation does apply some restrictions however. The APPLE-2 undulator is the most commonly used undulator for variable polarisation and as such is a mature solution, this type of undulator consists of planar magnet arrays above and below the electron beam. Finite element modelling has shown that a minimum wall thickness required to support the vessel under atmospheric load. For the elliptical case it is assumed that the full internal vertical aperture for the NLS undulators should be 7 mm, independent of the actual vessel geometry.

In addition to the APPLE-2 design there are three alternative options which deserve to be considered. The first is a variation on the APPLE-2 which is called APPLE-3 [6]. This design produces a higher helical field, by about a factor of 1.4, by partially surrounding a circular vacuum chamber with the permanent magnet blocks. The second option is called the Delta undulator [7]. In this device a circular vacuum vessel is totally surrounded by magnet blocks. This gives even higher field strengths, increasing the helical field by about a factor of 1.7 over the APPLE-2 device. This design also operates in fixed gap mode, making use of longitudinal array motion to both tune the photon energy and the
polarisation state. The key advantage of the increased magnetic field on axis is that the photon energy ranges required from the FELs could be obtained at a lower operating energy for the NLS.

The final option is to use standard linearly polarising planar undulators in the so-called crossed configuration [8]. In this case planar arrays generating a vertical field are followed by a second set generating a horizontal field. A phase shifter between the two sets allows the polarisation of the FEL radiation to be varied, on a relatively fast timescale.

A comparison has been made of the four possible undulator options and the results are summarised in Table 1. As well as the magnetic fields that are possible in the different polarisation states the minimum photon energy that can be generated by each undulator in circular polarisation mode is also given for a 2.25 GeV electron beam. The results suggest that the APPLE-2 would cover the narrowest photon range and that the Delta would cover the widest one. Clearly the Delta undulator is a very promising device but it is still in the early stages of development. Conversely, planar undulators are the lowest risk option from a magnetic point of view, but their use for generating arbitrary polarisation states introduces other complications in the FEL configuration [9]. To cover the required photon energy range of 430 to 1000 eV with FEL-3 is possible at 2.1 GeV with both the APPLE-3 and crossed planar undulators and at only 1.9 GeV with the Delta undulator. Any reduction in energy is economically attractive and so the three alternative schemes will all be carefully assessed in the near future.

Table 1: Comparison of the four undulator options assuming a 32 mm period. The Delta undulator fields have been calculated by Radia [10], the others are from empirical equations [6, 11, 12]

<table>
<thead>
<tr>
<th>Energy (eV) (circ. poln.)</th>
<th>APPLE-2</th>
<th>APPLE-3</th>
<th>Delta</th>
<th>Planar (Hybrid)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Field (T)</td>
<td>0.85</td>
<td>1.09</td>
<td>1.23</td>
<td>1.05</td>
</tr>
<tr>
<td>Helical Field (T)</td>
<td>0.51</td>
<td>0.68</td>
<td>0.86</td>
<td>NA</td>
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<tr>
<td>Horiz. Field (T)</td>
<td>0.63</td>
<td>0.87</td>
<td>1.23</td>
<td>1.05</td>
</tr>
<tr>
<td>Vessel Geometry</td>
<td>Elliptical</td>
<td>Circular</td>
<td>Circular</td>
<td>Elliptical</td>
</tr>
</tbody>
</table>

**SUMMARY**

Four options for the NLS FEL undulators have been considered. As part of the comparison a full assessment has been made of the impact of resistive wall wakefields, with a modelled start-to-end electron bunch, including the effect of vessel size and shape.

The results have shown that for the NLS there is negligible difference between circular and elliptical vessels with the same vertical apertures. Mechanically, the circular vessel offers a slight advantage in terms of required vessel wall thickness, though this is not of major significance overall.

Modelling of the FEL-3 power output has shown that an internal vertical aperture of 7 mm will give a ~10% loss in peak power in comparison with what is practically realisable. Direct comparison of the four undulator schemes using this internal aperture has shown the clear advantage of the Delta undulator over all the others. The electron energy of NLS could be reduced by ~15% if this undulator was adopted by the project. The impact of adopting this undulator for the NLS will be carefully assessed in the near future. The option of crossed planar undulators, which have the additional advantage of fast polarisation switching, will also be studied in the context of optimum configuration for the NLS, polarisation level vs output power, and higher harmonic polarisation.

Finally, the mitigation of wakefield effects with tapering or longer undulator sections will be studied as well as the impact of timing jitter and the inclusion of wake effects in the modulator sections.

**REFERENCES**